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On the Interaction between Small Decay, Agent Heterogeneity and Diameter of Minimal Strict Nash Networks in Two-way Flow Model: A Note

Banchongsan Charoensook

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By Banchongsan Charoensook, Adams College, Keimyung University

Summary

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Keywords: Network Formation, Strict Nash Network, Two-way Flow Network, Branching Network

JEL Classification: C72, D85

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Address for correspondence:
Banchongsan Charoensook
Adams College, Keimyung University
Department of International Business
Daegu, 42601
Republic of Korea
Email: 11596@gw.kmu.ac.kr

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Banchongsan Charoensook
Department of International Business
Adams College, Keimyung University
Republic of Korea
email: 11596@gw.kmu.ac.kr

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Abstract

Abstract. In this note, I study the roles of value heterogeneity - i.e., agents are heterogeneous in terms of values of nonrival information thay they possessed - in determining the shapes of two-way flow Strict Nash networks when small amount of decay is present. I do so by extending the two-way flow network with small decay of De Jaegher and Kamphorst (J ECON BEHAV ORGAN, 2015). Results of this extension shows that the effects of value heterogeneity on Strict Nash networks, when small decay is present, largely resemble the effects of heterogeneity in link formation cost - without decay - found in the literature. Another surprising finding is that value heterogeneity can extend the diameters of Strict Nash networks without changing any other properties. In the discussion section of this note, I relate this finding to two well-known concepts in the studies of social networks - small world and preferential attachment.

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1 Introduction

Nonrival information/resources refers to information/resources such that the one agent's access to the information does not negatively impact the benefits from consuming the same information/resources by another agent. Each piece of information that an agent possesses, though, is likely to vary in terms of value. Such variation in terms of information value can depend either on the identity of agent who possesses it (i.e., some agents may know useful information that has higher values than others. We call this partner heterogeneity) or the identity of agent who chooses to access it (i.e., the same information may have less value and hence less appreciated from the points of views of some agents compared to the points of views of other agents, or we call this player heterogeneity) or both (henceforth, I call this two-way heterogeneity).

In this note, I study how this heterogeneity in the value of nonrival information impacts the shapes of Strict Nash networks (SNNs, henceforth). I do so by extending the work of De Jaegher and Kamphorst (2015), which is a two-way flow model of network with small information decay. By allowing for the presence of value heterogeneity in this model, I study the changes in terms of shapes and other important properties of SNNs compared to original result of De Jaegher and Kamphorst (2015) (Proposition 1 and 2) in which value heterogeneity is absent as well as Galeotti, Goyal, and Kamphorst (2006) in which information decay is absent but value heterogeneity is present.

Within this literature, the studies on the role of agent heterogeneity in link formation cost have been quite extensive, both in terms of equilibrium characterization and existence of SNN [1]. Little is known, however, about the role of agent heterogeneity in information value [2]. To the knowledge of the author, the only finding in the literature about the role of agent heterogeneity in information value on two-way network is that it does not change the shape of SNN although disconnected components can emerge. This result is established as Proposition 3.1 in the work of Galeotti et al. (2006),

[1] See Galeotti et al. (2006), Billand, Bravard, and Sarangi (2011), Haller, Kamphorst, and Sarangi (2007), Billand, Bravard, and Sarangi (2012), Charoensook (2015), Charoensook (2019)

[2] While little is known about the role of value heterogeneity in Two-way flow network of nonrival information, there are quite some studies on the role of value heterogeneity in other types of network formation model. For works related to one-way flow network, see Billand, Bravard, and Sarangi (2013), Galeotti (2006), and Derks and Tennekes (2009). For works related to network formation model that requires mutual consent, see Persitz (2009) and Cofre (2016)

which assumes the absence of information decay. Intuitively, since there is no information decay, information transmission is perfect and hence every agent accumulates the same amount of information. Thus, heterogeneity in information value cannot cause agents in the same components to be different from each other. Of course, if information decay is present this line of reasonings can no longer holds. *This raises the question of the extent to which value heterogeneity can have impacts on SNN when information decay is present.* This present note, therefore, makes a small contribution to this insufficient body of literature by answering this question. It studies *the roles of agent heterogeneity in information value in the presence of small information decay rather than being completely absent* as in the previous literature, where the term 'small' here refers to the fact that no superfluous link is worth establishing by an agent [3]. Indeed, to my knowledge this note is the first paper in the literature of two-way flow network with nonrival information that reports that value heterogeneity has substantive effects on the shapes of Strict Nash networks.

This paper proceeds as follows. I introduce notations and the models in the next section. Then I establish the results concerning the equilibrium characterizations in the third section, where the three equilibrium characterizations here correspond to three types of agent heterogeneity - player heterogeneity, partner heterogeneity, and free-flowing heterogeneity. In this section, I also remark and explain why the roles of value heterogeneity when small decay is present do by and large resemble the roles of heterogeneity in link formation cost (without decay) reported in existing literature. In the fourth section, based upon the comparison between my results and the results of De Jaegher and Kamphorst (2015) that assumes value homogeneity, I point out some interestingly intricate interaction between value heterogeneity, small decay, and diameter of SNNs. To fast forward, I show that: (i) a balanced SNN is no longer resilient to the change to decay level (as decay level reaches zero), (ii) indeed, neither balanced no unbalanced, shorter or longer diameter SNN seems to be resilient to the change in decay level, and (iii) consequently, for the two-way flow model to have properties that

[3] As mentioned in De Jaegher and Kamphorst (2015) (whose model assumes value homogeneity) '...information decay has two effects. First, ex-ante homogeneous players become heterogeneous by their position in the network...Second, decay may give the individual player an incentive to sponsor links to players he is connected to, but indirectly'. my focus on small decay as opposed to all levels of decay, therefore, make possible to study the interaction between the first effect mentioned in De Jaegher and Kamphorst (2015) with value heterogeneity in isolation of the second effect. This is the rationale for the assumption of small decay in this paper

resemble small-world network and serves as a micro-foundation of preferential attachment, it has to be the case that heterogeneity among information value possessed by each agent is sufficiently small.

2 The Model

Since this work is an extension of De Jaegher and Kamphorst (2015)'s, most notations will also follow De Jaegher and Kamphorst (2015).

Link establishment and individual's strategy. Let $N = \{1, ..., n\}$ be the set of all agents. An agent $i \in N$ can form a link with another agent j without j's consent. ij denotes such a link. The set of all possible links that i forms is $L_i = \{ij : j \in N \backslash \{i\}\}$. Naturally, $g_i \subset L_i$ is a strategy of i and $g = \cup_{i \in N} g_i$ is a strategy profile. A strategy space G is , of course, the set of all possible g. Pictorially, a strategy profile g is also a network, where an arrow from agent i to j indicates that $ij \in g_i$.

Information flow. Let $N_i^S \{g\}$ denote the set of all agents i establishes a link with and let $\bar{N}_i^S \{g\}$ denote the set of all agents i establishes a link with or receives a link from. Let $\bar{ij}$, an undirected link, be a typical member of the set $\bar{N}_i^S \{g\}$. Since we assume that information flow is two-way, the collection of all undirected links, denoted by $\bar{g}$, represents the structure of information flow in a network. Specifically, information flows between i and j whenever there exists a path between i and j, which is defined as $P_{ij}(g) = \left\{\overline{i_0 i_1}, ..., \overline{i_{k-1} i_k}\right\} \subset \bar{g}$ such that $i_0 = i, i_k = j$. A shortest path between i and j is, of course, the path(s) between i and j with the least amount of links. A distance between i and j is defined as the amount of links of the shortest path(s). If $j = i$ then we assume, following the literature, that the distance of between i and himself is 0.

Value heterogeneity Let V_{ij} denote the value of information of j that arrives to i, given that the information flow is perfect. Let $\mathcal{V} = \{V_{ij}\}_{ij \in N \times N}$ be the value structure. $\mathcal{V}$ is said to satisfy value player heterogeneity if V_{ij} $= V_i$ for every agent i, which means that V_{ij} does not depend on the identity of j. Similarly $\mathcal{V}$ is said to satisfy value partner heterogeneity if $V_{ij} = V_j$ for every agent i, which means that V_{ij} does not depend on the identity of j. If there is no restriction on $\mathcal{V}$, we say that $\mathcal{V}$ is free-flowing. Onwards, for the sake of convenience I use $\mathcal{V}_{\text{player}}$, $\mathcal{V}_{\text{partner}}$, $\mathcal{V}_{\text{free}}$ to represent these three types of heterogeneity.

Cost heterogeneity Similarly, let c_{ij} denote the link formation that i bears to form a link with j. Let $\mathcal{C} = \{c_{ij}\}_{ij \in N \times N, i \neq j}$ be the cost structure. If $c_{ij} = c_i$ for every $i \in N$, then we say that $\mathcal{C}$ satisfies cost player heterogeneity. Similarly if $c_{ij} = c_j$ for every $i \in N$, then we say that $\mathcal{C}$ satisfies cost partner heterogeneity. If there is no restriction on $\mathcal{C}$, we say that $\mathcal{C}$ is free-flowing.

Information quantity Let the decay factor $\sigma \in [0, 1]$ represents the fact that V_{ij} decays per each link that information of j traverses to reach i. That is, if the distance between i and j is k then the information that i receives from j is $\sigma^k V_{ij}$, which is called ex-post value of information that i receives from j.

Small decay Following the assumption above and assuming an ij-path exists, an agent i can improve the flow of information from another agent j by establishing a link that leads to a shorter ij-path. Of course i has enough incentive to do so only if the benefit from so doing exceeds the link establishment cost c. However, if the decay is sufficiently small, i.e., σ is sufficiently close to 1, then the decay incurred by each path becomes nearly identical and the incentive to establish an extra path disappears. As a result, there is at most only one path between any pair of agents. This assumption is assumed in De Jaegher and Kamphorst (2015) and will be assumed throughout this paper.

Better-informed agent Let N_i^d be the set of all agents whose distance from i is d and N_i be the set of all agents whose path with i exists. We set $I_i(g) = \sum_{j \in N_i} \sigma^{d(i,j;g)} V_{ij}$ and call $I_i(g)$ total ex-post information that i received. Consider $M \subseteq N$ that is connected, ie., there is a path between any distinct pair of agents in M. Let $g_M = \{ij \in g : i, j \in M\}$. i is better-informed than j if $I_i(g_M) \geq I_j(g_M)$ and best-informed in g_M if $I_i(g_M) \geq I_j(g_M)$ for all $j \in M$. Let $A_{ij}(g)$ be the set of agents that i observes exclusively via the link ij. Of course, j has to be a best-informed agent in $A_{ij}(g)$ if i is to play his best response.

Similarly, we define total ex-post information that i' receives from the perspective of i as:

$$I_{i;i'}(g) = \sum_{k=0;k \neq i}^{n-1} \sigma^{d(i',k;g)} V_{ik}$$

Let $i', j' \in g' \subset g$ but $i \notin g'$. i' is said to be better-informed than j' from the perspective of i if $I_{i;i'}(g') > I_{i;j'}(g')$ and i' is best informed in g' from the perspective of i if $I_{i;i'}(g') \geq I_{i;j'}(g')$ for every $j \in N(g')$.

Network-related notations. Let $g' \subset g$ be a subnetwork of g. g' is said to be a component of g if there is a path between i and j for any $i, j\ inN(g')$ and no path between i and j whenever either $i \notin N(g')$ or $j \notin N(g')$ but not both. An agent who has no link with any other agent is called a singleton. Note that a singleton is also a component. Specifically we call a component that is a singleton an empty component. A network is connected if it has a unique non-empty component that contains all agents in the network. A non-empty component of a network or a network is minimal if there is at most one path among any pair of agents in the network. A minimally connected network is a rooted directed tree if every agent receives exactly one link except one agent that receives no link. The agent that receives no link is called root.

In a minimally connected network, let $g' \subset g$ be a subnetwork of g that is also minimally connected. Let $M \in N$ be the set of agents in g'. We say that $j \in M$ is the middle agent in g' if for every $k \neq j, k \in M$ it holds true that more than half of the players in M (including j and k themselves) are closer to j than k.

The payoffs. Let $V_i(g) = f(I_i(g))$ where $f' > 0$. The payoff of i :

$$U_i(g) = V_i(g) - \sum_{j \in N_i^S} c_{ij}.$$

A special case of this is the so-called linear payoff, which is:

$$\begin{aligned} U_i(g) &= I_i(g) - \sum_{j \in N_i^S} c_{ij} \\ &= \sum_{j \in N_i} \sigma^d(i, j; g) V_{ij} - \sum_{j \in N_i^S} c_{ij}. \end{aligned}$$

Strict Nash Networks. Consider a network g^* such that a strategy of i is $g_i^* \in g$. Let $g_{-i}^* = g^* \backslash g_i^*$ so that $g^* = g_i^* \in g \cup g_{-i}^*$. g_i^* is said to be a best response of i if $U_i(g^*) \geq U_i(g_i \cup g^* \backslash g_i^*)$ for every $g_i \neq g_i^*$. If the inequality is strict, then g_i^* is a unique best response of i. g is said to be a Nash network (NN, henceforth) if every agent chooses his best response. A Strict Nash network (SNN, henceforth) is a network such that every agent chooses his unique best response. In the main analysis section, SNN is used for equilibrium characterization rather than NN in order to allow for the comparison between our models, which assume value heterogeneity, and existing models that assumes cost heterogeneity such as Billand et al. (2011) and Galeotti et al. (2006).

2.1 Main Analysis: Equilibrium Characterization

In this section, we characterize SNNs given that value structure satisfies player heterogeneity, partner heterogeneity and free-flowing heterogeneity. Our motivation is to allow for the comparison with existing literature on the roles of heterogeneity in link formation cost that assumes likewise, namely (Galeotti et al. (2006) for player-based cost heterogeneity and free-flowing heterogeneity and Billand et al. (2011) for partner-based cost heterogeneity).

Proposition 1 (Partner Heterogeneity and Player Heterogeneity: Connectedness)**.** *If the value structure satisfies player heterogeneity or partner heterogeneity, SNN has a unique non-empty component. Moreover, in case of partner heterogeneity SNN is connected.*

Proof. [*Uniqueness of the non-empty component*] For $V_{ij} = V_j$ the proof is identical to Lemma 6 of De Jaegher and Kamphorst (2015). For $V_{ij} = V_i$ Lemma 6 of De Jaegher and Kamphorst (2015) needs the following modification. Recall from Section 2 that $I_{i;i'}(g)$ denotes the quantity of ex-post information arriving to i' in g from the perspective of i. Note that if we have i and j such that $\frac{V_i}{V_j} = \epsilon$ then we have $V_i = \epsilon V_j$ and $\epsilon I_{j;i'}(g) = I_{i;i'}(g)$. Now in Lemma 6 of De Jaegher and Kamphorst (2015), replace the inequalities in the fourth line by $I_{i;i'}(g \backslash ii') > I_{i;j'}(g) > I_{i;j'}(g \backslash jj')$ and the inequalities in the fifth line by $I_{j;j'}(g \backslash jj') > I_{j;i'}(g) > I_{j;i'}(g \backslash ii') \iff \epsilon I_{j;j'}(g \backslash jj') > \epsilon I_{j;i'}(g) > \epsilon I_{j;i'}(g \backslash ii') \iff I_{i;j'}(g \backslash jj') > \epsilon I_{j;i'}(g) > I_{i;i'}(g \backslash ii')$. A contradiction.

[*Connected/disconnectedness*] $V_{ij} = V_j$, the proof is identical to Lemma 5 of De Jaegher and Kamphorst (2015), allowing us to conclude that the unique non-empty component of SNN contains all agents. Note that for $V_{ij} = V_i$, the proof of Lemma 5 of De Jaegher and Kamphorst (2015) does not apply. Indeed, if there exists i such that $(n-1) V_i < c$ and $V_{i'} < c$ for every $i' \neq i$ and the payoff is linear, then obviously i is a singleton.

□

Next we further characterize the shapes of SNN for player heterogeneity case and partner heterogeneity case. Surprisingly, the shape of the unique non-empty component of SNN can be described in the same way as that of Proposition 1 of De Jaegher and Kamphorst (2015), which assumes value homogeneity [4].

[4] Actually, Example 3 of Charoensook (2019) also partially points out this fact. The difference is that Charoensook (2019) uses the term B_i and branching network, and Charoensook (2019) mentions only the case of partner heterogeneity.

Proposition 2 (Partner Heterogeneity and Pure Player Heterogeneity: Characterization)**.** *Let the value structure satisfies partner heterogeneity or player heterogeneity and given decay being sufficiently small, the unique non-empty component of SNN has similar characteristics with SNN that assumes value homogeneity as in Proposition 1 of De Jaegher and Kamphorst (2015), which we quote below:*

1. *g is a rooted directed tree with all links pointing away from its root: the unique non-recipient player. Each best-informed player in g is either the root player, or receives a link from the root player;*

2. *g is a directed tree with a unique multi-recipient player. Any link not received by this player points away from him. Moreover, this player is the unique best-informed player in g.*

Proof. Lemmata 7, 8 and 9 apply directly to the extension such that $V_{ij} = V_j$ or $V_{ij} = V_i$ without any modification. ☐

Why does it turn out that the shapes of SNN for the cases of player heterogeneity and partner heterogeneity are similar to the case of homogeneity as in De Jaegher and Kamphorst (2015)? First, let us compare player heterogeneity case with homogeneity case, both cases are actually similar in the sense that every agent i perceives every other agent as identical in terms of link formation cost and the level of decay factor associated with the link that he plans to establishes. Second, let us compare partner heterogeneity case with homogeneity case, both cases are also quite similar. Consider four agents i, i' and j. Let us assume that i and i' are not the in the same component that contains j. Then the quantities of information that i and i' would receive if they establish a link with j are identical since the assumption of partner heterogeneity necessitates that the identity of link sender is irrelevant. These similarities are the intuitions that explains this result. Naturally, if we allow for V_{ij} to depend on both i and j so that player heterogeneity and partner heterogeneity are violated then these lines of reasonings no longer apply. Clearly unlike the case of partner heterogeneity it becomes possible that i earns a large quantity of information as he forms a link with j while i' could earn just a little quantity of information even if the same agent j is the receiver of the link that he establishes.

While the Proposition 1 and 2 above tell us what kind of SNNs emerge given a value structure, it does not answer as to what the set of all possible SNNs are. Proposition 3 below answer this question.

Proposition 3 (Pure Partner Heterogeneity and Pure Player Heterogeneity: Reverse Characterization)**.** *Any minimally connected network whose properties are as either in (i) or (ii) described in Proposition 2 above can be supported as SNN by some $\mathcal{V}_{partner}$, c and σ.*

The proof of this proposition is driven by a lemma, which is stated below

Lemma 1. *In a network or subnetwork g such that $V_{ij} = V_j$ (V_{ij} is free-flowing). Any agent, say j, there exists $\bar{V}_j$ ($\bar{V}_{ij}$) such that j can be set as a unique best informed agent (from the perspective of i) by setting $V_j > \bar{V}_j$ ($V_{ij} > \bar{V}_{ij}$) while holding V_k (V_{ik}) constant for all $k \neq j, k \in g$ for all $0 \leq \sigma < 1$.*

proof of Lemma 1. For any i, k we have $\frac{\mathrm{d}I_j(g)}{\mathrm{d}V_j} = 1 > \frac{\mathrm{d}I_k(g)}{\mathrm{d}V_j} = \sigma^{d_{kj}}$. Therefore, $\bar{V}_j$ exists.

If $V_{ij} = V_i j$ then recall that an agent j's ex-post information in g from the perspective of i is $I_{i;j}(g) = \sum_{k=0;k\neq i}^{n-1} \sigma^{d(j,k;g)} V_{ik}$. Therefore, the proof is the same as the above paragraph except that we replace the above inequality by $\frac{\mathrm{d}I_{i;j}(g)}{\mathrm{d}V_{ij}} = 1 > \frac{\mathrm{d}I_{i;k}(g)}{\mathrm{d}V_{ij}} = \sigma^{d_{kj}}$. □

This lemma is intuitive. Simply put, it says that any agent can become a best informed agent if the value of this information is sufficiently high. Naturally, it follows that given a minimally connected network we can set any agent j to be best informed and well worth establishing a link with by setting V_j ($V_i j$) to be well above the link formation cost c as well and values of information of other agents. This is the intuition behind the proof of Proposition 3 below.

Proof of Proposition 3. [Proof for minimality and the existence of the cost range c] The proof follows from the proof of Proposition 2 of De Jaegher and Kamphorst (2015) except the last two lines, which are respect to the balancing condition which is not related to this paper.

Let us assume $\sigma < 1$, let us assume minimality for now. Later on we will prove it. For now we will prove only that every agent j that receives from i is a unique best informed agent in A_{ij}

[Rooted directed tree case]

First, note that the existence of σ is provided through Lemma 4 of De Jaegher. Next, we introduce the following notation. Let d denote the the distance between an agent and the agent i^*. Let i^d denote an agent whose distance from i^* is d. Let $\bar{d}$ denote the longest distance between i^* and an agent in this network. Note that since this network is branching rooted at

i^* we know that i^d receives exactly one link from i^{d-1} unless $d-1=0$ which means $i^{d-1}=i^*$. Thus, beginning from the distance for every $i^{\bar{d}-1}$ by Lemma 1 it suffices that we set $V_{i^{\bar{d}-1}} >> V_{i^{\bar{d}-1}}$ so that $i^{\bar{d}-2}$ finds that accessing $i^{\bar{d}-1}$ is a unique best informed agent in $A_{i^{\bar{d}-1}i^{\bar{d}-2}}$. This algorithm is repeated until the distance 1. The proof follows.

[Directed tree with a unique multi recipient Case] Let i^* be the unique multi receipient. The proof is nearly the same as the proof for the Rooted Directed Tree Case above. The only additional sentence is that we need to set $V_{i^*} >> V_j$ for every $j \neq i^*$. This warrants that those i^1 who are the periphery of i^* who access i^* finds that accessing i^* are their unique responses. The proof follows.

□

We compare the results of our equilibrium characterization in Proposition 1,2 and 3 above with those of other models in the literature as follows. Surpringly, quite some similarities are shared with those models that assume heterogeneity in link formation without decay. Put differently, the combination of value heterogeneity and information decay impacts SNN in the same way that the combination of link formation cost heterogeneity without decay does.

Remarks 1. *The effects of $V_{ij}=V_i$ and small decay (Proposition 1 and 2 in this paper) on SNNs resemble the effects of $c_{ij}=c_i$ without decay on SNNs (Proposition 3.2 in Galeotti et al. (2006)): they both result in the disconnectedness of SNNs, while the shapes of each non-empty component of SNNs are identical to the cases in which heterogeneity is absent.*

More specifically, the shape of non-empty component of SNN assuming $V_{ij}=V_i$ and small decay (Proposition 1 and 2 in this paper) is identical to value heterogeneity is absent and small decay is present (Proposition 1 in De Jaegher and Kamphorst (2015)). The only difference is that there can be isolate agents $V_{ij}=V_i$. This is why we conclude that $V_{ij}=V_i$ and small decay result in disconnectedness of SNN. This is analogous to the model of Galeotti et al. (2006) that assumes $c_{ij}=c_i$ but information decay is absent. This leads to SNN being disconnected center-sponsored star instead of a unique center-sponsored star that contains all agents, which is the result of the model that assumes homogeneity in link formation cost (Proposition K in Bala and Goyal (2000)).

Remarks 2. *The effects of $V_{ij}=V_j$ and small decay (Proposition 3 in this paper) on SNNs resemble the effects of $c_{ij}=c_j$ without decay on SNNs*

(Proposition 2 in Billand et al. (2011)). Indeed, the set of possible SNNs in Proposition 3 of this paper and Proposition 2 in Billand et al. (2011) are identical [5].

While Remark 2 above compared our results with that of Billand et al. (2011), it is also worth comparing our results in Proposition 2 and 3 with those in Proposition 1 and 2 of De Jaegher and Kamphorst (2015) since it assumes small decay without agent heterogeneity. Quite surprisingly while Proposition 2 in our paper states that the shapes of SNNs in our model are described precisely in the same way as the shapes of SNNs in Proposition 1 of De Jaegher and Kamphorst (2015), our Proposition 3 shows that the diameter of SNN when value heterogeneity is present tends to be much longer than the case in which value heterogeneity is absent as in De Jaegher and Kamphorst (2015). Indeed, a corollary of our Proposition 3 is that SNN can be a line network, which is the network with the longest diameter possible. This is a contrast to the result of De Jaegher and Kamphorst (2015) since they mentioned on page 225 that "...the diameter of SNNs will be small compared to the total population size..." We formally state this comparison as another remark below.

Remarks 3. *A notable effect of $V_{ij} = V_j$ on SNN, compared to the case in which value heterogeneity is absent as in De Jaegher and Kamphorst (2015), is that the diameter of SNN becomes relatively longer. Indeed, SNN can also be a line.*

Finally, we turn to provide a full characterization in the general case of value heterogeneity. Proposition 4 below shows that in such a case any minimal network can be SNN.

Proposition 4. *Any minimal network can be supported as SNN by some $\sigma, c,$ and $\mathcal{V}$.*

Proof. First, for now we assume that $V_{ij} = 0$ whenever i and and j are not from the same component. Thus, for any $c > 0$ there is no incentive for the link ij or ji to be established.

Now g^c be a typical non-empty component of g. for every ij in g, similar to the proof of Proposition 3 and by Lemma 1, it suffices that we set $V_{ij} >> V_{ik}$ for every $k \in N(A_{ij}(g^c))$. This allows j to be a unique best informed agent in $A_{ij}(g^c)$. The proof follows.

[5] while our description of SNNs and the description of SNNs in Billand et al. (2011) appear to be different, they are actually identical. This is a result of Lemma 3 of Charoensook (2019)

Next, the existence c and σ that allow each non-empty component to be minimally connected is proven in Proposition 2 of De Jaegher and Kamphorst (2015). Finally, we relax the assumption (recall the first sentence of this proof) that $V_{ij} = 0$ whenever i and j are not from the same component by noting that that $f'()$ is continuous in V_{ij}. Consequently there exists $\bar{\epsilon} > 0$ such that if $0 < V_{ij} < \bar{\epsilon}$ for all i, j not in the same component then i and j have no incentive to establish a link with one another. □

Remarks 4. *The effects of the combination of $\mathcal{V}$ that is free flowing and small decay on SNN resemble the effects of heterogeneity in link formation cost that is free flowing without decay. Indeed, the classes of network that can be supported as SNN in Proposition 4 above and Proposition L of Galeotti et al. (2006), which assumes general heterogenety in link formation cost, are identical.*

Overall, our analyses above show that the combination of value heterogeneity and small decay impacts SNN in the same way as cost heterogeneity (without decay) does. While this may appear to be surprising, the primary intuition is quite simple. From value heterogeneity perspective, an agent is a superior partner choice, and indeed the best choice of partner, if the total ex-post value of information that he posseses is *highest* among other agents. Similar, from cost heterogeneity perspective, an agent is the best choice of partner if the link formation cost such that he is the link receiver is the *lowest* among other agents. Thus, value heterogeneity and cost heterogeneity are much like reverse mirror images of each other. Of course, this line of reasoning here holds true only if small decay is assumed to be present together with value heterogeneity. Otherwise, there is no agent who is superior as a partner choice from value heterogeneity perspective since information quantity possessed by each agent becomes identical due to the absence of decay.

3 Value Heterogeneity, decay, and network diameter: relations between the three

3.1 Motivating Examples and Remarks

In this subsection, my intention is to examine an intricate interplay of three elements - level of small decay, diameter of SNN, and value heterogeneity. A primary motivation is, as stated in Remark X, once when partner-driven value heterogeneity replaces value homogeneity diameters of SNNs can become very long and indeed a line network can also be SNN. Another primary motivation is that the equilibrium characterization in Proposition K does not answer what happens to SNN as decay diminishes, i.e., $\sigma \to 1$. This means that our results cannot be compared directly to the result in Lemma L and Prop X of De Jaegher, which asserts that a balanced SNN is resilient to the change in the change in σ as $\sigma \to 1$. In this section, we also wish to find out that this very resilience of balanced SNN holds true in the extension that replaces value homogeneity with partner-driver value heterogeneity. We begin to explore these issues by stating one remark below.

Remarks 5. *Assuming $\mathcal{V}$ satisfies partner heterogeneity and small decay is present, consider a minimally connected network or subnetwork containing two agents, say, j and k. Assuming value partner heterogeneity, let N_1 denote all agents except j and k that are closer to j than k but are not located within the undirected path between j and k, N_2 denotes all agents except j and k that are closer to k than j but are not located in the undirected path between j and k, N_3 denote all agents except j and k that are closer to j than k and are located in the undirected path between j and k, N_4 denotes all agent except j and k that are closer to k than j and are located in the undirected path between j and k. j is better informed than k iff:*

$$
\begin{aligned}
&V_j + \sigma^{d_{j,k}} V_k + \sum_{l \in N_1} V_l \sigma^{d_{j,l}} + \sum_{l \in N_2} V_l \sigma^{d_{j,l}} + \sum_{l \in N_3} V_l \sigma^{d_{j,l}} + \sum_{l \in N_3} V_l \sigma^{d_{j,l}} \\
&\quad > V_k + \sigma^{d_{j,k}} V_j + \sum_{l \in = N_1} V_l \sigma^{d_{k,l}} + \sum_{l \in N_2} V_l \sigma^{d_{k,l}} + \sum_{l \in N_{btw\dot{}jk\dot{}cl\dot{}j}} V_l \sigma^{d_{k,l}} + \sum_{l \in N_4} V_l \sigma^{d_{k,l}}
\end{aligned}
$$

Note that for $l \in N_1$ $d_{j,l} + d_{l,k} = d_{j,k}$ and for $l \in N_2$ $d_{j,k} + d_{k,l} = d_{j,l}$ so that we have:

$$
\begin{aligned}
&V_j \left(1 - \sigma^{d_{j,k}}\right) + \sum_{l \in N_1} V_l \left(1 - \sigma^{d_{j,k}}\right) \sigma^{d_{j,l}} + \sum_{l \in N_3} V_l \left(\sigma^{d_{l,j}} - \sigma^{d_{l,k}}\right) \\
&\quad > V_k \left(1 - \sigma^{d_{j,k}}\right) + \sum_{l \in N_2} V_l \left(1 - \sigma^{d_{j,k}}\right) \sigma^{d_{k,l}} + \sum_{l \in N_4} V_l \left(\sigma^{d_{l,k}} - \sigma^{d_{l,j}}\right)
\end{aligned}
$$

iff:

$$
\begin{aligned}
V_j > {} & V_k \left(1 - \sigma^{d_{j,k}}\right) + \sum_{l \in N_2} V_l \sigma^{d_{k,l}} + \sum_{l \in N_4} V_l \left(\sigma^{d_{l,k}} - \sigma^{d_{l,j}}\right) \left(1 - \sigma^{d_{j,k}}\right)^{-1} \\
& - \sum_{l \in N_1} V_l \sigma^{d_{j,l}} + \sum_{l \in N_3} V_l \left(\sigma^{d_{l,j}} - \sigma^{d_{l,k}}\right) \left(1 - \sigma^{d_{j,k}}\right)^{-1}
\end{aligned}
$$

Now let us put on a few notation to help shorten the above inequality

$$V_j > \overbrace{V_k\left(1-\sigma^{d_{j,k}}\right)+\sum_{l\in N_2}V_l\sigma^{d_{k,l}}+\sum_{l\in N_4}V_l\left(\sigma^{d_{l,k}}-\sigma^{d_{l,j}}\right)\left(1-\sigma^{d_{j,k}}\right)^{-1}}^{\textcircled{A}}$$

$$-\overbrace{\sum_{l\in N_1}V_l\sigma^{d_{j,l}}+\sum_{l\in N_3}V_l\left(\sigma^{d_{l,j}}-\sigma^{d_{l,k}}\right)\left(1-\sigma^{d_{j,k}}\right)^{-1}}^{\textcircled{B}}$$

Note that we do not know if Ⓐ − Ⓑ is decreasing in σ as $\sigma \to 1$, since the first differentiation of Ⓐ − Ⓑ with respect to σ depends on σ as well as V_l , $l \neq j$. Thus, given $\mathcal{V}$ and σ, it is possible that $V_j >$ Ⓐ − Ⓑ but $V_j < \lim_{\sigma\to 1}$ Ⓐ − Ⓑ. Equivalently (since this is an iff condition), this means that that j is better informed than k for a given $\mathcal{V}, \sigma$. But k becomes better than j as $\sigma \to 1$. In example 1 and 2 below we use this result to show that for the same set $\mathcal{V}, c$ it is possible that for a given level of σ a relative shorter diameter network is SNN while a relative longer diameter network cannot be SNN. But as $\sigma \to 1$ the situation gets to be reversed: a relative longer diameter network is SNN while a relative shorter diameter network cannot be SNN.

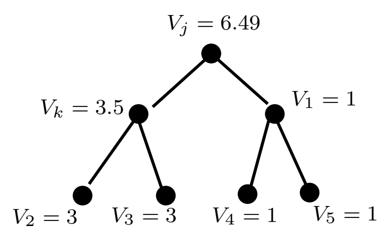

Figure 1: Example 1

Example 1. *Consider the (undirected) network in Figure 1 above. Note that j is the middle agent while k is not. Between j and k, which agents are better informed? We claim that j be better informed than k given the support* $\mathcal{V} = \{V_j = 6.49, V_k = 3.5, V_1 = 1, V_2 = 3, V_3 = 3, V_4 = 1, V_5 = 1\}, c = 0.98, \sigma = 0.99$. *However, as $\sigma \to 1$ the converse holds, ie., k becomes better than j. We prove this claim using the last inequality in the above Remarks 1, j is better than k if and only if:*

$$V_j > V_k + \sigma V_2 + \sigma V_3 - \left(\sigma V_1 + \sigma^2 V_4 + \sigma^2 V_5\right) \tag{1a}$$

substituting the numbers (where $\sigma = 0.99$) we have

$$6.49 > 6.484 \tag{1b}$$

Hence, it holds true that j is better informed than k for this supporting parametrs. However, if we try to find the limit of LHS and RHS as $\sigma \to 1$ we have:

$$6.49 < 6.5 \tag{1c}$$

As a result, there exists $\underline{\sigma} > 0.99$ such that k is better informed than j for all $\sigma \in (\underline{\sigma}, 1)$. Indeed, a tedious computation similar to the above will show that j is best informed in this network for $\sigma = 0.99$ but k becomes the best informed agent for as $\sigma \to 1$

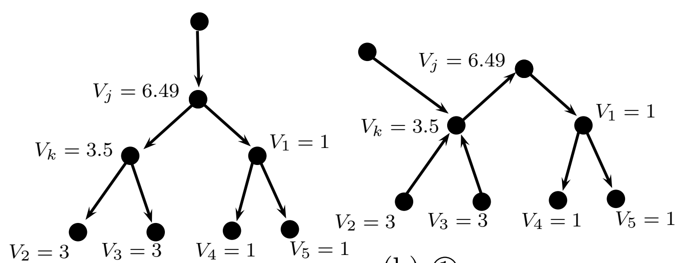

Figure 2: Networks with six agents

Example 2. *As a continuation of the above Example 1, consider the support $\mathcal{V} = \{V_i = 1, V_j = 6.49, V_k = 3.5, V_1 = 1, V_2 = 3, V_3 = 3, V_4 = 1, V_5 = 1\}, c = 0.98, \sigma = 0.99$. Note that the only difference between the support in this example and the support in Example 1 is the addition of $V_i = 1$. Obviously, the balanced network g_1 above is SNN given this support. However, as $\sigma \to 1$ g_1 ceases being SNN. Instead g_2 becomes SNN. This fact requires that j is best informed $g_1 - ij$ given this support and k is best informed in $g_2 - ik$ as $\sigma \to 1$ respectively. Indeed, the Example 1 above supports this claim, since the undirected networks $\bar{g_1} - ij = \bar{g_2} - ik$ are identical to the undirected network in Example 1.*

Using the same line of reasonings, it is quite straightforward to shows the opposite of Examples 1 and 2 above can also take place. That is, a longer, unbalanced network is SNN for a σ but is not resilient to $\sigma \to 1$ while a balanced SNN is. We show these examples below.

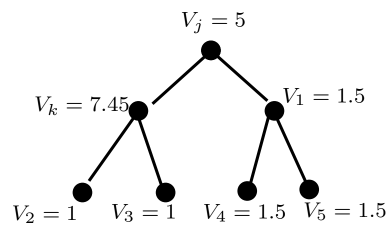

Figure 3: XXYY.

Example 3. *Consider the undirected network similar to Example 1 above, but we change the support to be* $\mathcal{V} = \{V_j = 5, V_k = 7.45, V_1 = 1.5, V_2 = 1, V_3 = 1, V_4 = 1.5, V_5 = 1.5\}, c = 0.98, \sigma = 0.99$. *A similar to computation to Example 1 will show that* k *is best informed in this undirected network given this support. However, as* $\sigma \to 1$ j *is better informed in this network .*

Example 4. *As a continuation of the above Example 4, using the same analogy as Example 2 it is straightforward to show that* g_2 *is SNN but* g_1 *is not given the support* $\mathcal{V} = \{V_1 = 1, V_j = 5, V_k = 7.45, V_1 = 1.5, V_2 = 1, V_3 = 1, V_4 = 1.5, V_5 = 1.5\}, c = 0.98, \sigma = 0.99$. *However, as* $\sigma \to 1$ g_1 *becomes SNN and* g_2 *ceases to be SNN.*

3.2 Discussion 1: Does small decay - as opposed to high decay - favor high diameter SNN?

First, let us recall that small decay in the absence of value heterogeneity seems to favor SNNs that are of high diameters. According to De Jaegher and Kamphorst (2015) "...especially the smaller amount of decay can result in high diameter networks. It is rather for a larger amounts of decay that all high diameter networks will fail to be SNNs.' (De Jaegher and Kamphorst (2015) p. 225).' Once value heterogeneity is present as in this article, does this fact continues to hold true?

Our examples in the above section show that the answer is no. Indeed, as $\sigma \to 1$ it could be the case that SNN's diameter got longer or shorter. In the example 1 and 2, it turn out that SNN became longer, an evidence that supports the statement of De Jaegher and Kamphorst (2015) as shown in the above paragraph. However, in the example 3 and 4, the result is the opposite in the sense that SNN became short, an evidence that is against the above statement of De Jaegher and Kamphorst (2015). In conclusion, once value heterogeneity is present, its interaction with small decay neither favors a longer diameter SNN or shorter diameter SNN.

What intuitions drive this result? The answers are quite simple. In the absence of value heterogeneity, the level of information arrives to each agent depends only on their location in the network. Thus, a middle agent turns out to be the best informed agent as $\sigma \to 1$, since the middle agent has a position that is 'optimal' in the sense that he is not so far away from any other agents. However, once the value heterogeneity is present it is no longer the case that only what matters is the location. Instead, it is the location, the decay level, and the value of information that an agent possesses as well as the value of information of agents that are his neighbors that altogether determine the identity of best informed agent in a network. Consider the undirected network in the Example 1, we can observe that the agents who are closer to j than k - namely 3, 4 and 5 - have information values that are quite lower than the agents who are closer to k - namely 1 and 2. However, the information that j possesses has a much higher value to k's information. Thus, for decay level that is sufficiently high j is best informed thanks to the value of the information that he possesses. However, as $\sigma \to 1$, k become best informed due to the fact that values of information of agents that are just one-link away from him - namely j, 1 and 2 - are quite high. Of course, using the same line of reasonings it could also happen that j becomes better informed than k as $\sigma \to 1$. This is seen in the Example 3. [6]

3.3 Discussion 2: Does small decay resemble the small-world phenomena?

According to De Jaegher and Kamphorst (2015), the role of small decay in their model provides a micro-foundation to the small-world properties of network because '...preferential attachment is key to understanding small world networks. Preferential attachment means that new players are more likely to form links with players who have many links than with players who have few links. The two-way ow model with decay offers a micro-foundation for preferential attachment. Given that you care about the distance to other players, it is typically more attractive to sponsor a link to a player with many links, than to a player with few links'. Our finding is that the argument that an agent would find that an agent with many links is more attractive as a

[6] Another interesting point is that this result is remarkably different from other works that assume agent heterogeneity such as the in-dept analysis of the insider-outsider model as in Billand, Bravard, and Sarangi (2010), which is a model that assumes heterogeneity in link formation cost (as opposed to value in our model). In this model, they find that SNN has a shorter diameter as $\sigma \to 1$. Specifically a minimally connected SNN in the insider-outsider model can have diameter of at most 5. But as $\sigma \to 1$ SNN has a diameter of at most 3.

partner than a player with few links is no longer valid if value heterogeneity is assumed. Simply put, even if an agent has only one link or even no link but the value of information that he possesses is extremely high, he remains more attractive than another agent who has multiple links with agents whose information values are relatively much smaller.

Indeed, this very tradeoff - choosing between an agents with less links but higher information values versus an agent with multiple links but smaller information values - leads to the key feature in our model that SNN can have a very long diameter and indeed SNN can even a line. As such, once value heterogeneity is assumed, SNN will no longer have a feature that resembles the small world phenomena, which is such that 'the network diameter is of an order substantially smaller than that of the population.' (De Jaegher and Kamphorst (2015), p.219) Hence, our results show that for the two-way flow network with small decay to serve as a micro-foundation for preferential attachment and also have small-world properties that resemble real-world network, it has to be the case that the values of information/resources possessed by each agent are not very different from each other.

4 Conclusion

In this note, I study the roles of value heterogeneity on the properties of Strict Nash networks when the presence of small decay is assumed. I provided equilibrium characterizations based on three types of heterogeneity - player heterogeneity, partner heterogeneity, and free-flowing heterogeneity. My results show that the effects of combination of value heterogeneity and small decay on SNNs resemble those of link formation cost found in the literature. I also draw a comparison between the effects of combination of value heterogeneity and small decay on SNNs with the effects of small decay alone on SNN as in De Jaegher and Kamphorst (2015). Interestingly, once the assumption that information value depends on the identity of agent who possesses it replaces the assumption of value homogeneity in De Jaegher and Kamphorst (2015), the only property of SNN that changes is that SNNs now can have long diameters compared to SNNs under the case of value homogeneity found in De Jaegher and Kamphorst (2015). Consequently, in the discussion section above I conclude that for the two-way flow model with small decay to serve as a microfoundation of preferential attachment, which helps understand small world networks, the extent to which value heterogeneity is present in the network has to be sufficiently small.

This note can be extended in many ways, depending on how value heterogeneity is specified. Interesting extensions would be to assume some specific forms of value heterogeneity that are meant to reflect some intuitive ideas. For example, one could assume a value structure such that $V_{ij} = V_{ji} = V_H$ whenever i and j belong to the same group and $V_{ij} = V_{ji} = V_L < V_H$ whenever i and j does not belong to the same group. That is, information has a high value only to agents who are from the same group where the same group here could consist of agents who share the same interests. This way, we have a value structure that mirrors the 'insider-outsider' model pioneered by Galeotti et al. (2006), which is such that link formation cost is lower whenever it is between two agents from the same group. Another example is the value structure that mirrors the situation in which one group of agents has ex-ante power over all other agents, so that a link with an agent from this 'power group' always yields a higher value than a link with an agent that is not from this group. This form of value structure has been studied in-depth by Persitz (2009) in the setting where link formation requires mutual consent. I expect that it would also be interesting to study the same form of value structure in the setting where information is nonrival and hence link formation is unilateral, as in this note.

Another interesting branch of extension is related the recent research pioneered by Olaizola and Valenciano (2015) and Olaizola and Valenciano (2018). These authors propose innovative models that unify the two seminal models - one is Bala and Goyal (2000) whose link formation is unilateral and the other one is Jackson and Wolinsky (1996) whose link formation is bilateral - as one. Their models allow for both cases to happen in the network, given that a link that is unilaterally formed is weaker, incurring higher degree of information decay, than a link that is bilaterally formed. It would be interesting to understand the effects of value heterogeneity on equilibrium networks in this model, since this would provide insights on the roles of value heterogeneity in a more general setting of network formation than what this note provides.

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Fondazione Eni Enrico Mattei
Corso Magenta 63, Milano – Italia

Tel. +39 02.520.36934
Fax. +39.02.520.36946

E-mail: letter@feem.it
www.feem.it