1. Introduction

The empirical analysis of the economic interactions between factors of production, output and corresponding prices has received much attention over the last two decades.

Although the many contributions in this area differ substantially in the functional specification of the technology, the inclusion of any dynamics and the treatment of expectations formation, they all agree on the neoclassical principle of a representative optimizing firm and typically use theory-based structural equation models (SEM). Among the different alternatives proposed in the literature to model and evaluate dynamic factor demands within the SEM approach, dynamic duality theory is a relatively new and promising tool which was developed in the early 1980's (McLaren and Cooper, 1980; Epstein, 1981) and applied in the fields of production analysis (Epstein and Denny, 1983; Chang and Stefanou, 1988; Bernstein and Nadiri, 1989; Manera, 1994) and agricultural economics (Vasavada and Chambers, 1986; Howard and Shumway, 1988; Luh and Stefanou, 1991, 1996; Fernandez-Cornejo, Gempesaw II, Elterich and Stefanou, 1992). Within this framework, full consistency with the adjustment cost (ADC) scheme and the underlying dynamic optimization problem of the firm is ensured by the dual relationship existing between the firm's technology and its intertemporal value function. Based on this relationship, it is possible to derive closed-form factor demand equations using a generalization of Shephard's (Hotelling)'s lemma and avoiding the explicit solution of the optimal control problem.

A popular alternative to SEM is given by the vector autoregression (VAR) methodology, which can be interpreted as a response to one major weakness of SEM, namely the a priori division between endogenous and exogenous variables. Within this approach, the process of describing the complex relationships between economic variables starts from the formulation of an unrestricted VAR model, where each series is explained as a function of its own history only, of the lagged values of the remaining series, and possibly some deterministic components (constants, trends, seasonals and dummies). The lag length is taken to be large enough to capture the temporal properties of the variables and treat disturbances as innovations. Since many macroeconomic time series exhibit non-stationary characteristics and the distinction between endogenous and (weakly) exogenous variables is often arbitrary, it would be desirable in applied research to use an approach which could be easily adapted to model integrated variables and to test for exogeneity. The VAR methodology provides the researcher with a useful tool to analyze short-run (SR) as well as long-run (LR), or cointegration, relationships among the non-stationary variables (Johansen, 1988). The issue of conditioning upon a particular set of variables can be addressed by adapting Johansen's (1992) analysis to the (weak) exogeneity case (Urbain, 1992; Boswijk, 1993).

Although not numerous, various attempts to link the SEM approach with VAR analysis can be found in the recent literature on factor demands (Engsted and Haldrup, 1994, 1999). Little, if any, effort has been devoted to comparing these alternative approaches when firms are assumed to face a multi-factor technology.

This paper bridges this gap. We illustrate how the SEM and the VAR approaches can both represent valid alternatives to model systems of dynamic factor demands. Moreover, we show how to apply the methodologies to estimate dynamic factor demands derived from a cost-minimizing capital-labour-energy-materials (KLEM) technology with ADC on the quasi-fixed capital factor, using annual observations on the Italian total manufacturing sector. Then, we discuss how to use both models to calculate some widely accepted indicators of the production structure of an economic sector, such as price and quantity elasticities, and alternative measures of ADC. In particular, we propose and discuss some theoretical and empirical justifications of the differences between observed elasticities, measures of ADC, and the assumption of exogeneity of output and/or input prices. Finally, we provide some suggestions for the applied researcher interested in modeling factor demand systems.

The paper is organized as follows. Section 2 is devoted to a brief outline of the SEM and VAR approaches. Section 3 contains an analysis of the statistical behaviour of the economic time series, together with the econometric specification and estimation of the SEM and the VAR models. Section 4 is dedicated to the practical use of both models of factor demands. Section 5 gives some concluding comments.

2. Modeling dynamic factor demands using SEM and VAR

2.1. *The SEM approach*

In the SEM approach, structural equations originate from a fully specified, possibly non-linear model of the economy, where suitable functional forms for the fundamentals of the model (i.e. preferences and technologies) have been selected. Optimization of some underlying objective function implies decision rules (i.e. reduced form equations) for the endogenous variables of the model, which can be written in terms of the exogenous predetermined variables and a set of non-linear cross-equation restrictions. Since regressors could be correlated with the errors, a non-linear, instrumental variable system estimator is generally needed in order to avoid the sumultaneous equation bias. For example, in factor demand systems, the presence of the level change of the quasi-fixed factor as a regressor in the equations for the variable inputs and the endogeneity of input prices and/or output are common sources of simultaneity bias.

More formally, consider the (non-linear) simultaneous equation model defined by the following system of n factor demand equations in implicit form (see Bowden and Turkington, 1984, p. 185):

(1) $f_i(y_t, z_t, \xi_i) = u_{it}$, i=1,…,n; t=1,…,T

where y_t is an nx1 vector of endogenous variables, z_t is a sx1 vector of exogenous variables, and ξ_i is a vector of parameters. Not all of the elements of y_t and z_t may actually appear in the arguments of each f_i. We define an nx1 error vector u_t as $(u_{1t},u_{2t},\ldots,u_{nt})'$. Assume that the vectors u_t, t=1,…,T, are independently and identically normally distributed with zero mean vector and covariance matrix Σ.

In factor demand analysis, errors across input equations are expected to be contemporaneously correlated, implying that the nxn error variance-covariance matrix Σ would be non-diagonal (Berndt, 1991, p. 463). In addition, the presence of u_t is generally justified in two ways (Hayashi, 2000, p. 301). One is to admit that firms make random errors in choosing their cost-minimizing input combinations. The second is to allow the intercept coefficients to be stochastic and vary across firms. In this latter case the constant intercept in the i-th equation would be the mean of the random intercept, and the error term would be the deviation of the random intercept from its mean.

Define $f_t = (f_{1t},f_{2t},\ldots,f_{nt})$ and $f_{it} = f_i(y_{1t},y_{2t},\ldots,y_{nt},z_{1t},\ldots,z_{st},\xi_i)$. Assume that partial derivatives exist and are continuous and that $\partial f_t / \partial y_t'$ and $\sum_{t=1}^{T} f_t f_t'$ are non-singular. System (1) can be written in stacked form as f(ξ) = f(y, x, ξ) = u, where $\xi = (\xi'_1,\xi'_2,\ldots,\xi'_n)'$ and the first T elements of the stacked vectors f or u correspond to the first equation, the second T elements to the second equation, and so on.

Noting that $\mathrm{Cov(u)} = \Sigma \otimes I_T$, an instrumental variable estimator is the value of ξ which minimizes φ(ξ) = f(ξ)'Pf(ξ) for some suitable choice of the matrix P. Assume there is a matrix V of instruments of order Txq, with q ≥ dim(ξ). Variables in V may not coincide with the exogenous variables that appear originally in the arguments of f.

The Non-linear Three-Stage Least Squares estimator (NL3SLS) (see Jorgenson and Laffont, 1974; Berndt, Hall, Hall and Hausman, 1974; Amemiya, 1997) is defined as the value of ξ that minimizes φ(ξ) when $P = \Sigma^{-1} \otimes V \left(V'V\right)^{-1} V'$.

Although the NL3SLS estimator is asymptotically less efficient than the Maximum Likelihood estimator, it is more robust against non-normality. Denote by G the data matrix $\partial f / \partial \xi'$ corresponding to derivatives of the functions f_i with respect to ξ_i, and define G_0 as the value of G at the true value $\xi = \xi_0$. Under suitable regularity conditions, the NL3SLS estimator is consistent and asymptotically normal with limiting covariance matrix $\left(\mathrm{plim}\ \frac{1}{\mathrm{T}} G_0' P G_0\right)^{-1}$ (Amemiya, 1977, p. 965). Amemiya (1985, p. 256) points out that non-linearity generally helps identification. For example, in a non-linear model the number of excluded exogenous variables in a given equation needs not to be greater than or equal to the number of parameters of the same equation. In addition, one sufficient condition for identifiability is that the limiting matrix $\mathrm{plim}\ \frac{1}{\mathrm{T}} G_0' P G_0$ is non-singular.

Factor demand models are often characterized by the presence of cross-equation restrictions (e.g. symmetry, homogeneity, monotonicity and concavity restrictions). The NL3SLS estimator can be easily accommodated to deal with constraints among the parameters. If, for example, the same parameters ξ_i appear in different equations, it is always possible to express each ξ_i as a function of θ, $\xi_i(\theta)$, where the number of elements in θ is less than those in ξ. Then, the inverse of the estimated asymptotic variance-covariance matrix has to be premultiplied by ∂ξ'/∂θ and postmultiplied by ∂ξ/∂θ' (Amemiya, 1977, p. 401).

Many estimated factor demand systems have demonstrated to be affected by residual autocorrelation. A simple way to deal with this problem is to extend the structure of the errors to a vector autoregressive process (Berndt, 1991, p. 477). For example, let assume that $u_t = \Phi u_{t-1} + e_t$, with e_t being a vector of independently and identically distributed errors, and Φ a nxn non-diagonal, asymmetric autocovariance matrix consisting in n^2 parameters. Write system (1) as $f'_t = u_t$. Taking into account the first-order autocorrelation structure of the errors, the system becomes:

$$f'_t = \Phi f'_{t-1} + e_t.$$

This system can be estimated by using the NL3SLS estimator discussed above. Since the number of parameters to be estimated has increased by n^2, even for small systems it is easy to run out of degrees of freedom. In this situation, restrictions can be imposed on matrix Φ, such as Φ diagonal or $\Phi = \phi I_n$, where ϕ indicates a correlation coefficient which is common across equations. All these restrictions, which assume that autocorrelation affects the equations of the system in very particular ways, can also be tested via Wald-type tests (Berndt, 1991, pag. 466).

In this paper we model and evaluate SEM (1) using dynamic duality. Within this approach, the relationship between the firm's technology and its intertemporal value function ensures full consistency of the model with the ADL scheme and the optimization problem of the firm, as well as the possibility of deriving closed-form factor demand equations via a simple generalization of Shephard's lemma.

Let the firm's technology be represented by the production function:

$$(2)\quad Q = \varphi(VF, FF, GI, t)$$

where Q is scalar output (or, equivalently, Q is a ν_0x1 vector with ν_0=1), VF is a ν_1x1 vector of variable inputs, FF is a ν_2x1 vector of quasi-fixed inputs, GI is a ν_2x1 vector of gross investment in the quasi-fixed factors, and t is time. The inclusion of time as an explicit argument in the production function captures the advancement in technology (Luh and Stefanou, 1996, p. 992). Moreover, notice that all variables are functions of time. GI as an argument of (2) accounts for the presence of internal ADC, brought about by changes in the level of capital stocks FF. The production function $\varphi(\cdot)$ is increasing in VF, FF and t, decreasing in GI, and concave in VF, FF, GI. Assuming that the firm minimizes the present value of its future costs at initial time t_0 under static price and output expectations, the objective function can be written as:

$$(3)\quad \min_{VF,GI} \int_{t_0}^{\infty} e^{-rt}(\hat{VP}' \cdot VF + \hat{FP}' \cdot GI)dt$$

subject to

$$(4)\quad \begin{aligned} GI &= \frac{dFF}{dt} + D \cdot FF \\ FF(t_0) &= FF_0 \end{aligned}$$

where $\hat{VP}$ is a ν_1x1 vector of prices of the variable inputs, $\hat{FP}$ is a ν_2x1 vector of prices for the quasi-fixed inputs, $\frac{dFF}{dt}$ is a ν_2x1 vector of net investment in the quasi-fixed factors, r is a constant interest rate, and D is a ν_2xν_2 diagonal matrix of constant depreciation rates. The assumption of time-invariant interest and depreciation rates is common to the vast majority of applications of intertemporal duality theory and can be rationalized using the continual replanning argument (see Galeotti, 1996, for a complete survey on the existing literature). In this way, r is absorbed into the functional form for the intertemporal value function.

Inverting the production function (2) with respect to, say, VF_1, yields the factor requirement function for VF_1, which is dual to the normalized restricted cost function:

$$(5)\quad C(VP, FF, GI, Q, t) = VP' \cdot VF$$

with $VP = \hat{VP} / \hat{VP}_1$ and C(·) is normalized for $\hat{VP}_1$. It is easy to show that problem (3) can be suitably rewritten as:

$$(6)\quad J(t_0, FF_0, RP, VP, Q, t) = \min_{GI} \int_{t_0}^{\infty} e^{-rt}\left[C(VP, FF, GI, Q, t) + RP' \cdot FF\right]dt$$

subject to (4), where $RP = (r + D) \cdot FP$, $FP = \hat{FP} / \hat{FP}_1$, is the rental price of FF normalized by $\hat{VP}_1$.

The dynamic duality theory (Epstein, 1981) defines, in close analogy to the static case, a formal relation between a given technology, represented here by the dual cost function C(·), and the intertemporal value function J(·), which is the solution to (6). The general form of the Hamilton-Jacobi (HJ) equation for problem (6) at t_0 is given by (Kamien and Schwartz, 1991, p. 260) as:

$$-J_{t_0}(t_0, FF_0, RP, VP, Q, t) = \min_{GI}\{e^{-rt_0}\left[C(VP, FF_0, GI, Q, t) + RP' \cdot FF_0\right] + J'_{FF_0}(.)(GI - D \cdot FF_0) + J_t(.)\}$$

with $J_\psi(\cdot)$ indicating the first-order derivative of J(·) with respect to variable ψ, and $J_{\psi,\omega}(\cdot)$ being the second-order derivative of J(·) with respect to variables ψ and ω. Define:

$$J\left(t_0, FF_0, RP, VP, Q, t\right) \equiv e^{-rt_0}\tilde{J}\left(FF_0, RP, VP, Q, t\right),$$

where $\tilde{J}(.) = \min_{GI}\int_{t_0}^{\infty} e^{-r(t-t_0)}\left[C(.) + RP' \cdot FF\right] dt$, subject to equation (4). Using these last two expressions we obtain: $-J_{t_0}(.) = re^{-rt_0}\tilde{J}(.)$; $J_{FF_0}(.) = e^{-rt_0}\tilde{J}_{FF_0}(.)$; $J_t(.) = e^{-rt_0}\tilde{J}_t(.)$. Substituting into the HJ equation from problem (6) and multiplying both sides by e^{rt_0} yields:

(7) $$r\tilde{J}(.) = \min_{GI}\left[C(.) + RP' \cdot FF + \tilde{J}'_{FF_0}(.)(GI - D \cdot FF) + \tilde{J}_t\right].$$

The problem dual to (7) is:

(8) $$C^{\circ}(.) = \max_{RP}\left[r\tilde{J}(.) - RP' \cdot FF - \tilde{J}'_{FF_0}(.)\left(GI - D \cdot FF\right) - \tilde{J}_t(.)\right].$$

Applying the usual first-order necessary conditions for a maximum, one obtains:

(9) $$C^{\circ}_{RP}(.) = r\tilde{J}_{RP}(.) - FF - \tilde{J}'_{FF,RP}(.)(GI - D \cdot FF) - \tilde{J}_{t,RP}(.) = 0$$

which leads to the investment equation:

(10) $$GI^* = \left[\tilde{J}_{FF,RP}(.)\right]^{-1}\left[r\tilde{J}_{RP}(.) - FF - \tilde{J}_{t,RP}(.)\right] + D \cdot FF.$$

The (ν_1-1) variable input demand equations VF_{-1} (i.e. the vector of remaining variable factors, once VF_1 has been chosen as numeraire) can be obtained by taking first derivatives of (8) with respect to VP, after substituting (10) into (8):

$$(11)\ VF_{-1}^{*} = r\tilde{J}_{VP}(.) - \tilde{J}'_{FF,VP}(.)(GI^{*} - D \cdot FF) - \tilde{J}_{t,VP}(.)$$

whereas the demand equation for the variable input whose price has been chosen as numeraire can be obtained substituting (11) into (5):

$$(12)\ VF_{1}^{*} = r\tilde{J}(.) - RP' \cdot FF - \tilde{J}'_{FF}(.)\left(GI^{*} - D \cdot FF\right) - \tilde{J}_{t}(.) - VP' \cdot VF_{-1}^{*}.$$

Equations (10)-(12) represent the analogue of Shephard's lemma and provide a straightforward procedure for generating dynamic factor demands which can be jointly estimated.

In the empirical application, capital is assumed to be the only quasi-fixed factor, whereas labour, energy and materials are variable inputs. Capital follows a symmetric adjustment path towards its steady-state level. Standard assumptions are made on the ADC on the quasi-fixed factor, which are internal, convex and non-separable. Static expectations over relative factor prices and output are assumed. Finally, production factors are hypothesized to be exchanged in competitive markets. In this way, the firm purchases inputs at their market prices which, from the firm's viewpoint, are all exogenous.

In order to estimate the model, we characterize the intertemporal value function by the following quadratic form, although alternative parametrizations have been proposed in the applied literature (see, e.g., Howard and Shumway, 1988 and Luh and Stefanou, 1991):

$$\begin{aligned}(13)\ \tilde{J} = {} & a_0 + a_k sq_k + a_u np_k + a_y sq_y + a_e np_e + a_m np_m + a_t t \\ & + \frac{1}{2} a_{kk} sq_k^2 + \frac{1}{a_{ku}} sq_k np_k + a_{ky} sq_k sq_y + a_{ke} sq_k np_e + a_{km} sq_k np_m + a_{kt} sq_k t \\ & + \frac{1}{2} a_{uu} np_k^2 + a_{uy} np_k sq_y + a_{ue} np_k np_e + a_{um} np_k np_m + a_{ut} np_k t \\ & + a_{yy} sq_y^2 + a_{ye} sq_y np_e + a_{ym} sq_y np_m + a_{yt} sq_y t \\ & + \frac{1}{2} a_{ee} np_e^2 + a_{em} np_e np_m + a_{et} np_e t + \frac{1}{2} a_{mm} np_m^2 + a_{mt} np_m t\ .\end{aligned}$$

The variables involved in equation (13) are: capital stock (q_k), labour (q_l), energy (q_e), materials (q_m), net investment (q_{ni}), output (q_y), rental price of capital (p_k), price of labour (p_l), price of energy (p_e), price of materials (p_m), and time trend t. An "s" (or "n") at the beginning of a series name means that the series has been "scaled" (or "normalized" by the price of labour, p_l).

The reciprocal of a_{ku} appears in the quadratic form in order to reduce the nonlinearity of the investment equation, as suggested by Epstein (1981). Some peculiarities of the quadratic functional form are empirically relevant (see Galeotti, 1996, pp. 445-446). First, the quadratic linear homogenous cost function is non-nested with its non-homogeneous counterpart. This forces the researcher to choose one variable input as numeraire. Being the resulting demand function for the numeraire input different from those for other variable inputs, the empirical findings are not invariant to the choice of the numeraire input. Second, the quadratic cost function satisfies the curvature properties globally, as its Hessian matrix is constant, independent of the specific sample of data used in the empirical investigation. Finally, a cost function (optimal value function) specified with a quadratic functional form is self-dual, that is it can be solved analytically for the associated quadratic production function (cost function) and vice versa.

The expressions for $\tilde{J}_{FF,RP}(\cdot)$ and $\tilde{J}_{RP}(\cdot)$ are obtained from (13) and, upon substitution into (12), lead to the specification of the following investment equation:

(14) $$q_{ni}^* = a_{ku}\left[r\left(a_u + \frac{1}{a_{ku}} sq_k + a_{uu} np_k + a_{ue} np_e + a_{um} np_m + a_{uy} sq_y + a_{ut} t\right) - sq_k\right]$$

where q^*_{ni} denotes net investment. Notice that (14) is a flexible accelerator model, that is:

(15) $$q_{ni}^* = \lambda(sq_k^* - sq_k)$$

where:

(16) $$\lambda = -(r - a_{ku})$$

and

$$(17)\quad sq_k^* = \left[\frac{ra_{ku}}{(a_{ku}-r)}\right](a_u + a_{uu}np_k + a_{ue}np_e + a_{um}np_m + a_{uy}sq_y + a_{ut}t)$$

is the steady-state level of sq_k. The existence of a steady state requires that sq^*_k be positive. Stability of the adjustment path is assured if λ in (16) lies in between zero and one. Deriving expressions for $\tilde{J}_\psi(\cdot)$ and $\tilde{J}_{FF,\psi}(\cdot)$, ψ=np_e,np_m, from (13), and upon substitution into (11), the demand equations for the variable inputs energy and materials are obtained:

$$(18)\quad sq_e^* = r(a_e + a_{ke}sq_k + a_{ue}np_k + a_{ee}np_e + a_{em}np_m + a_{ye}sq_y + a_{et}t) - a_{ke}q_{ni}^*$$

and

$$(19)\quad sq_m^* = r(a_m + a_{km}sq_k + a_{um}np_k + a_{em}np_e + a_{mm}np_m + a_{ym}sq_y + a_{mt}t) - a_{km}q_{ni}^* .$$

Finally, the labour demand equation sq^*_l can be obtained using (12), together with (18) and (19):

$$\begin{aligned}(20)\quad sq_l^* = r\Big(&a_0 + a_k sq_k + a_u np_k + a_y sq_y + a_e np_e + a_m np_m + a_t t \\ &+ \frac{1}{2}a_{kk}sq_k^2 + \frac{1}{a_{ku}}sq_k np_k + a_{ky}sq_k sq_y + a_{ke}sq_k np_e + a_{km}sq_k np_m \\ &+ a_{kt}sq_k t + \frac{1}{2}a_{uu}np_k^2 + a_{uy}np_k sq_y + a_{ue}np_k np_e + a_{um}np_k np_m \\ &+ a_{ut}np_k t + a_{yy}sq_y^2 + a_{ye}sq_y np_e + a_{ym}sq_y np_m + a_{yt}sq_y t \\ &+ \frac{1}{2}a_{ee}np_e^2 + a_{em}np_e np_m + a_{et}np_e t + \frac{1}{2}a_{mm}np_m^2 + a_{mt}np_m t\Big) \\ &- np_k sq_k \\ &- \left(a_k + a_{kk}sq_k + \frac{1}{a_{ku}}np_k + a_{ky}sq_y + a_{ke}np_e + a_{km}np_m + a_{kt}t\right)q_{ni}^* \\ &- np_e sq_e^* - np_m sq_m^* .\end{aligned}$$

The SEM is then composed by four equations, one for each of the four endogenous variables q_{ni}, sq_e, sq_m and sq_l. The regressors of the equation of net investment are given by three deterministic components (a constant term, a linear time trend, a dummy variable for the year 1975), the quantity and normalized price of the quasi-fixed factor capital (sq_k and np_k), the normalized prices of the variable factors energy and materials (np_e and np_m), as well as scalar output sq_y (i.e. 8 regressors). Those eight regressors appear also in all the remaining three equations. More precisely, the equations for energy and materials include q_{ni} as an additional regressor (i.e. 9 regressors each). The labour equation exhibits, as additional regressors, q_{ni} and the squares and cross products among sq_k, np_k, np_e, np_m, sq_y and the time trend (the quadratic time trend term is not included in the specification, given the absence of a non-linear trend in the variables) (i.e. 29 regressors).

2.2. *The VAR approach*

Within the SEM approach described in Section 2.1, the behavioural equations of the relevant economic variables (factor demands, share systems) are generally derived as mathematical solutions of the deterministic optimizing problem of the representative firm. Random disturbances are introduced only at the estimation stage in order to embed the deterministic system of equations into a stochastic framework. In this way, the presence of random errors is difficult to justify, since it is hardly coherent with the optimization model (see McElroy, 1987, for a critical discussion of this issue). For example, if one assumes that disturbances are due to firm's random error in solving its cost minimization problem, then actual total costs should be not as low as what is prescribed by the cost function. Moreover, if the intercept parameter in factor demands is assumed to be random across firms, then also the cost function should be treated as stochastic (Hayashi, 2000, p. 301).

The VAR approach overcomes those problems by directly starting from the specification of an appropriate stochastic framework. The variables of an economic system are interrelated in a complex way, where non-stationarities, dynamics and specific events (e.g. temporary and/or permanent shocks) play a crucial role. The process generating the data is not known to the investigator and it can be described by the joint distribution function $D(X \mid X_0, \Theta)$, where the distribution of the mxT matrix $X^T{}_1$ containing T observations for each of the m variables is conditional on the mx1 vector of starting values X_0 and on a vector of unknown parameters Θ. Assuming that the mx1 vector of variables X_t, t=1,...,T, is non-stationary as a result of the presence of deterministic (e.g. linear or quadratic trends) as well as stochastic components (e.g. integrated variables), a model that incorporates both types of processes is the following unrestricted VAR:

(21) $A(L)X_t = \mu DET_t + \varepsilon_t$

where ε_t is a mx1 vector of error terms independently and identically normally distributed with zero mean vector and covariance matrix Σ; DET_t is a s_0x1 vector of deterministic components (i.e. constant term, time trend, impulse and/or step dummy variables capturing temporary shocks and/or permanent regime shifts); t=1,...,T is a time trend; A(L) is a p-th order matrix polynomial in the lag operator L, with $A_0=I_m$.

If X_t is an integrated vector of order one (I(1)), unrestricted VAR models like (21) can be formulated in first differences. However, if the variables are linked by some linear combinations which are stationary (cointegrating relationships), differencing will produce a loss of LR information. An alternative representation which distinguishes between SR and LR responses is the VAR-ECM model (Johansen, 1988):

(22) $$\Delta X_t = \sum_{i=1}^{p-1} \Pi_i \Delta X_{t-i} + \Pi X_{t-p} + \mu DET_t + \varepsilon_t$$

where $\Delta \equiv 1-L$, $\Pi_i = (-I_m+A_1+...+A_i)$ is the i-th interim multiplier, and $\Pi=(-I_m+A_1+...+A_p)$ is the matrix of static LR equilibria. Notice that equation (22) can be obtained from equation (21) by adding X_{t-1}, X_{t-2}, …, X_{t-p} and A_1X_{t-2}, A_2X_{t-3}, …, $A_{p-1}X_{t-p}$ to both sides of equation (21) (Charemza and Deadman, 1992, pp. 196-7). If ρ = rank(Π), with $0<\rho<m$, there exist ρ linear combinations of X_t that are I(0) (cointegrating, or LR relationships) and $m-\rho$ linear combinations of X_t which act as common stochastic trends (driving variables). In this case $\Pi=\alpha\beta'$, where both α and β are $m\times\rho$ matrices of rank ρ. The columns of β are formed by the coefficients of the ρ cointegrating vectors, so that the linear combinations $\beta' X_t$ are I(0), whereas the rows of α give the weights (loadings) attached to each cointegrating vector. A procedure to empirically assess the rank of Π has been developed by Johansen (1988). The null hypothesis of ρ being at most ρ^* (H_0: $\rho\leq\rho^*$) can be tested against two alternatives, the first one asserting that ρ is equal to p, the autoregressive order of the VAR (H_1: ρ=p, trace test), the second one assuming that ρ is equal to ρ^*+1 (H_2: $\rho=\rho^*+1$, maximum eigenvalue test). In both cases, the relevant asymptotic distributions are non-standard (Osterwald-Lenum, 1992). Once the rank of Π has been determined, it is then possible to obtain maximum likelihood estimates of α and β. Notice, in passing, that α and β are not unique, which means that some restrictions may be needed to achieve LR identification and provide α and β with a plausible economic interpretation. In addition, the validity of the procedure outlined so far depends on the correct specification of the unrestricted VAR in (21).

The VAR approach allows the investigator to tackle in a very direct way two important problems in economic modeling, namely (weak) exogeneity of a subset of regressors and encompassing.

In order to discuss the first, write the m-vector X_t as $X_t = (Z_t, Y_t)$, where Z_t and Y_t are vectors of dimensions sx1 and nx1, respectively. Then, partition α, Π_i, μ_0, μ_1, μ_2, ε_t and Σ conformably, that is $\alpha = (\alpha_Z, \alpha_Y)$, $\Pi_i = \begin{pmatrix} \Pi_{iZZ} & \Pi_{iZY} \\ \Pi_{iYZ} & \Pi_{iYY} \end{pmatrix}$, i=1,…,p-1, $\varepsilon_t = (\varepsilon_{Yt}, \varepsilon_{Zt})$ and $\Sigma = \begin{pmatrix} \Sigma_{ZZ} & \Sigma_{ZY} \\ \Sigma_{YZ} & \Sigma_{YY} \end{pmatrix}$. Related to (22) (and assuming μ=0), we can distinguish between a conditional unrestricted VAR-ECM model:

$$(23)\ \Delta Y_t = \Pi_0 \Delta Z_t + \alpha_{YZ}\beta' X_{t-p} + \sum_{i=1}^{p-1}(\Pi_{iYZ} - \Pi_0\Pi_{iZZ})\Delta Z_{t-i} + \sum_{i=1}^{p-1}(\Pi_{iYY} - \Pi_0\Pi_{iZY})\Delta Y_{t-i} + \varepsilon_{YZt}$$

and a marginal unrestricted VAR-ECM model:

$$\Delta Z_t = \alpha_Z \beta' X_{t-p} + \sum_{i=1}^{p-1}\Pi_{iZZ}\Delta Z_{t-i} + \sum_{i=1}^{p-1}\Pi_{iZY}\Delta Y_{t-i} + \varepsilon_{Zt}$$

where $\Pi_0 = \Sigma_{ZY}\Sigma_{ZZ}^{-1}$, $\alpha_{YZ} = \alpha_Y - \Pi_0\alpha_Z$, and $\varepsilon_{YZt} = \varepsilon_{Yt} - \Pi_0\varepsilon_{Zt}$ (Charemza and Deadman, 1992, pp. 260-1). A necessary and sufficient condition for Z_t to be weakly exogenous for α and β is $\alpha_Z = 0$ (Johansen, 1992; Urbain, 1992). Given this condition, efficient inference can be conducted directly on the conditional unrestricted VAR-ECM model in (23).

The notion of encompassing can be summarized as follows. Suppose there are two competing models, M_i and M_j, both nested within M_c, where M_c is the composite model formed by the explanatory variables in M_i augmented by the explanatory variables in M_j which do not appear already in M_i. Then M_i encompasses M_j ($M_i \mathrm{E} M_j$) if and only if M_i parsimoniously encompasses M_j ($M_i \mathrm{E}_p M_j$) (see Mizon, 1984; Hendry and Richard, 1989, p. 409). In this case M_i is a valid simplification of M_c and it summarizes all relevant features of both M_c and M_j. If we move to the multivariate context, testing if a particular structural equation model parsimoniously encompasses a statistically adequate VAR corresponds to testing the validity of the over-identifying restrictions imposed by the structural model on the VAR and it can be done using a standard likelihood ratio test.

2.3. *VAR-ECM and SEM: a unifying framework*

Recall that X_t is a mx1 vector of I(1) variables, with m=n+s, n=$\nu_1+\nu_2$ and s=$s_0+s_1+s_2$. Define with W_t a s_3x1 vector of (additional) exogenous variables. If we exclude the deterministic components in order to simply the notation and we limit the autoregressive component to one lag, a VAR(1) assumes the following form:

$$X_t = A_1 X_{t-1} + BW_t + u_t \,,$$

where A_1 and B are coefficient matrices of dimensions mxm and mxs_3, respectively; u_t is a mx1 vector of errors, whose distribution is multivariate normal with zero vector mean and covariance matrix Σ.

If ρ cointegrating vectors are present among the m variables X_t, the VAR(1) model has the ECM representation:

$$\Delta X_t = \alpha\beta' X_{t-1} + \Gamma \Delta W_t + u_t \,,$$

where α is the mxρ matrix of coefficients representing the speed of adjustment of the system to the ρ long-run equilibria $\beta' X_{t-1}$; β is the mxρ matrix of LR coefficients forming the cointegrating equations $\beta' X_{t-1}$; Γ is a mxs_3 matrix of parameters.

Partition now the vector X_t in the two subvectors Y_t and Z_t. Y_t is a nx1 vector of endogenous variables (where ν_1 indicates the number of variable factors and ν_2 is the number of quasi-fixed factors), whereas Z_t is a sx1 vector of exogenous variables (where s_0 indicates the number of deterministic components; s_1 is the number of normalized variable factor prices and scalar output; s_2 is the number of quasi-fixed factor prices and quantities). Assume, for simplicity, the presence of one quasi-fixed factor only (i.e. v_2=1), and denote net investment in that quasi-fixed factor with $y_{(1)t}$. Assume also that the s_3x1 vector of additional exogenous variables W_t is formed by the squares and cross product among output, factor prices, linear trend and the quantity of the quasi-fixed input (with the exclusion of the quadratic trend term).

The SEM can be written as:

$$(24)\quad \begin{bmatrix} y_{(1)t} \\ y_{(2)t} \\ \vdots \\ y_{(n-1)t} \\ y_{(n)t} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_{n-1} \\ C_n \end{bmatrix} \begin{bmatrix} [Z_t] & 0 & 0 & \cdots & 0 \\ 0 & \begin{bmatrix} Z_t \\ y_{(1)t} \end{bmatrix} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & \cdots & \begin{bmatrix} Z_t \\ y_{(1)t} \end{bmatrix} & 0 \\ 0 & 0 & 0 & 0 & \begin{bmatrix} Z_t \\ y_{(1)t} \\ \vdots \\ y_{(n-1)t} \\ W_t \end{bmatrix} \end{bmatrix} + \begin{bmatrix} u_{(1)t} \\ u_{(2)t} \\ \vdots \\ u_{(n-1)t} \\ u_{(n)t} \end{bmatrix}$$

where C_j, j=1,…,n, are vectors of parameters. In particular, the dimension of each vector is as follows: C_1 is 1xs, C_j, j=2,…,n-1, are 1x(s+1), and C_n is 1x(s+1+s_3). The SEM specification is completed by the long-run equation of the quasi-fixed factor $y_{(1)t}$, $y_{(1)t} = \lambda\gamma' Z_t$. Coefficient λ measures the speed of adjustment of $y_{(1)t}$ to its LR level, while the sx1 vector of parameters γ indicates the weights associated with the Z variables in the LR relation.

The Unrestricted Reduced Form (URF) corresponding to the SEM is:

$$(25)\quad \begin{bmatrix} y_{(1)t} \\ y_{(2)t} \\ \vdots \\ y_{(n-1)t} \\ y_{(n)t} \end{bmatrix} = \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_{n-1} \\ \tilde{C}_n \end{bmatrix} \begin{bmatrix} [Z_t] & 0 & 0 & \cdots & 0 \\ 0 & [Z_t] & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & 0 \\ 0 & 0 & \cdots & [Z_t] & 0 \\ 0 & 0 & 0 & 0 & \begin{bmatrix} Z_t \\ W_t \end{bmatrix} \end{bmatrix} + d \begin{bmatrix} y_{(1)t-1} \\ y_{(2)t-1} \\ \vdots \\ y_{(n-1)t-1} \\ y_{(n)t-1} \end{bmatrix} + \begin{bmatrix} u_{(1)t} \\ u_{(2)t} \\ \vdots \\ u_{(n-1)t} \\ u_{(n)t} \end{bmatrix}.$$

Variables and parameter vectors have the same meaning as in the SEM. The only differences are given by: the dimension of C_j, j=1,…,n-1, which are now 1xs vectors; the dimension of $\tilde{C}_n$, which is now a $1x(s+s_3)$ vector; the presence of d, which is a nx1 vector of parameters. Notice that parameters in $\tilde{C}_n$ are, in general, (non-)linear functions of the parameters C_j, j=1,…,n-1. Moreover, the number of over-restrictions imposed by the SEM on the URF is $(n+s_3)$, that is the sum between the dimensions of W_t and Y_t. These restrictions can be tested using a standard likelihood ratio statistic.

In the empirical specification of our SEM, we assume that: $v_0 = 1$ (scalar output sq_y); $v_1 = 3$ (variables factors labour, energy and materials sq_l, sq_e, sq_m); $v_2 = 1$ (quasi-fixed input capital sq_k); $n=v_1+v_2=4$. Moreover, $s_0 = 3$ (constant, linear time trend and dummy variable for year 1975); $s_1 = v_0 + (v_1-1) + v_2 = 4$ (output sq_y, normalized prices of energy and materials np_e, np_m, and normalized price of capital np_k); $s_2 = 2v_2 = 2$ (normalized price and quantity of the quasi-fixed input capital); $s_3 = 20$ (squares and cross-products among sq_k, np_k, np_e, np_m, sq_y, and the linear time trend, with the exclusion of the quadratic trend term).

3. Empirical evidence

3.1. *Data analysis*

The KLEM data set used in this empirical study is given by the annual time series of capital stock (q_k), labour (q_l), energy (q_e) and materials (q_m), together with net investment (q_{ni}), output (q_y), rental price of capital (p_k), price of labour (p_l), price of energy (p_e), price of materials (p_m), and constant interest and depreciation rates for the Italian manufacturing sector over the period 1954-1983. This particular data set is widely used in many applied studies on the production structure of the Italian economy (see Manera, 1994, for detailed references), since it represents the first attempt to reconstruct annual time series on the relevant variables using a common methodology (see Heimler and Milana, 1984, for details). Unfortunately, subsequent changes in the way the original variables are collected by the Italian institute of statistics have prevented us from updating this data set to more recent years. All series are expressed in logarithms. A "s" (or "n") at the beginning of a series name means that the series has been "scaled" by 10^{-4} (or "normalized" by the price of labour, p_l) before taking the logarithmic transformation. The series of manufacturing output, energy, materials and labour inputs and price indexes are taken from Heimler and Milana (1985), whereas the series of gross fixed capital and investment, disaggregated by type of capital goods and sectors, can be found in Rosa and Siesto (1985). The aggregate depreciation rate δ is constant and set equal to 0.049 on the basis of average lives published in Rosa (1979, p. 8), whereas the constant aggregate interest rate r is 0.077. The series used to obtain the investment goods price index are those of Heimler and Milana (1984), and the rental price of capital $p_{k,t}$ has been computed by applying Christensen and Jorgenson's (1969, p. 302) well-known formula:

$$p_{k,t} = rp_{gi,t-1} + \delta p_{gi,t} - \left(p_{gi,t} - p_{gi,t-1} \right)$$

where p_{gi} is the gross price of investment goods. Since the available data refer to the end-of-year value of capital stock, capital enters the model with a one-period lag.

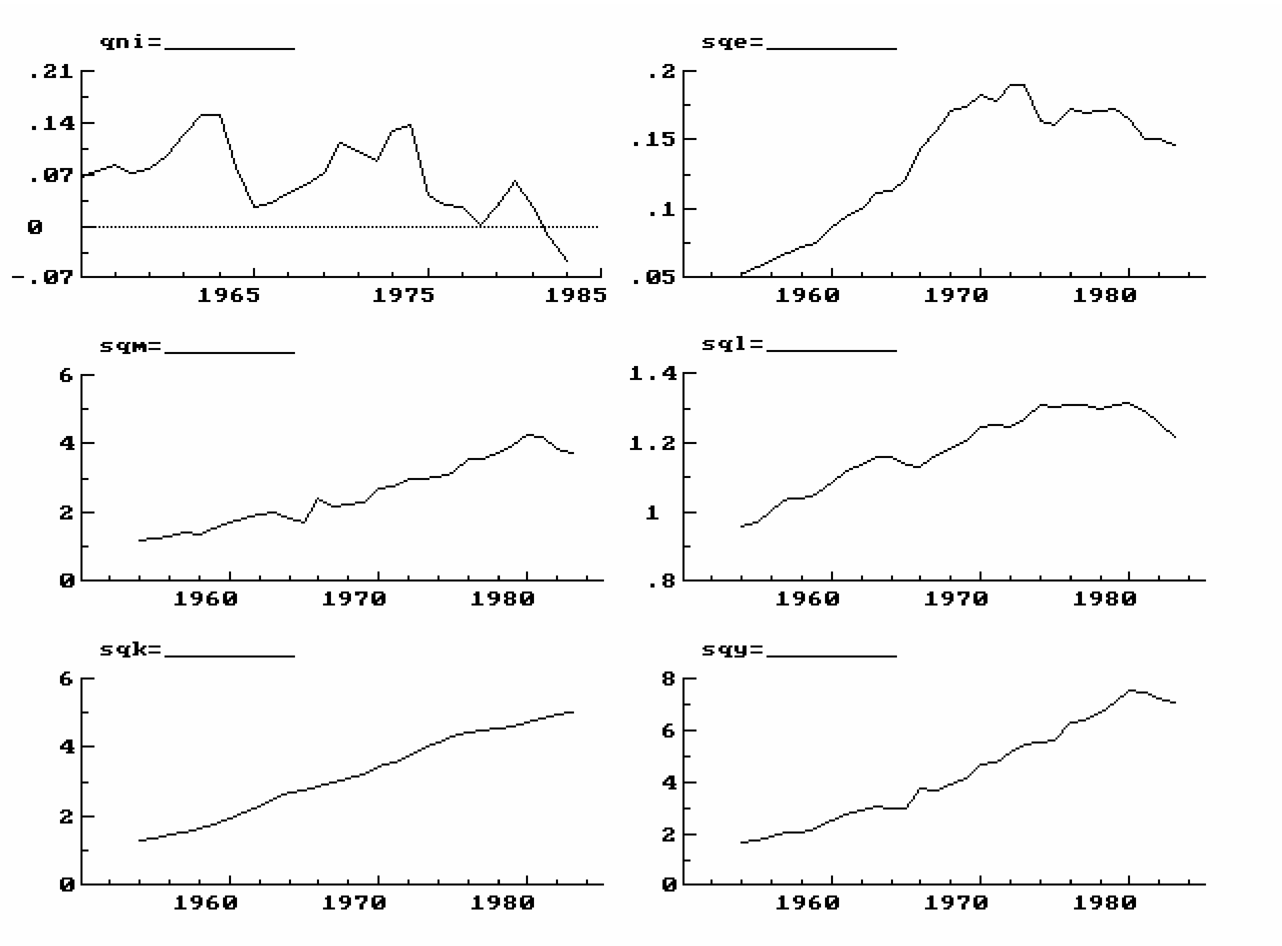

Figure 1. Levels of net investment (q_{ni}), energy (sq_e), materials (sq_m), labour (sq_l), capital stock (sq_k) and output (sq_y)

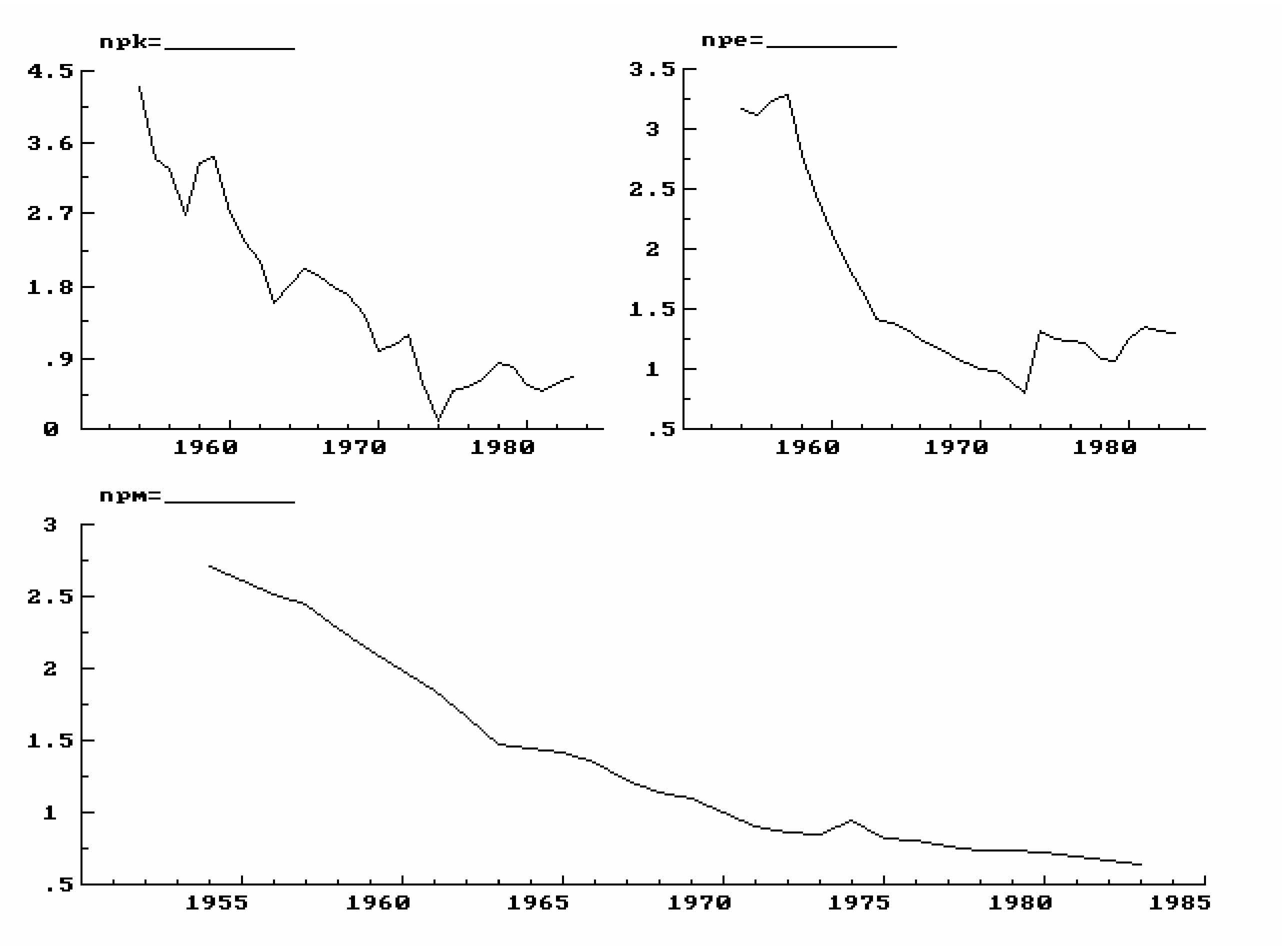

Figure 2. Levels of the normalized rental price of capital (np_k), price of energy (np_e) and price of materials (np_m)

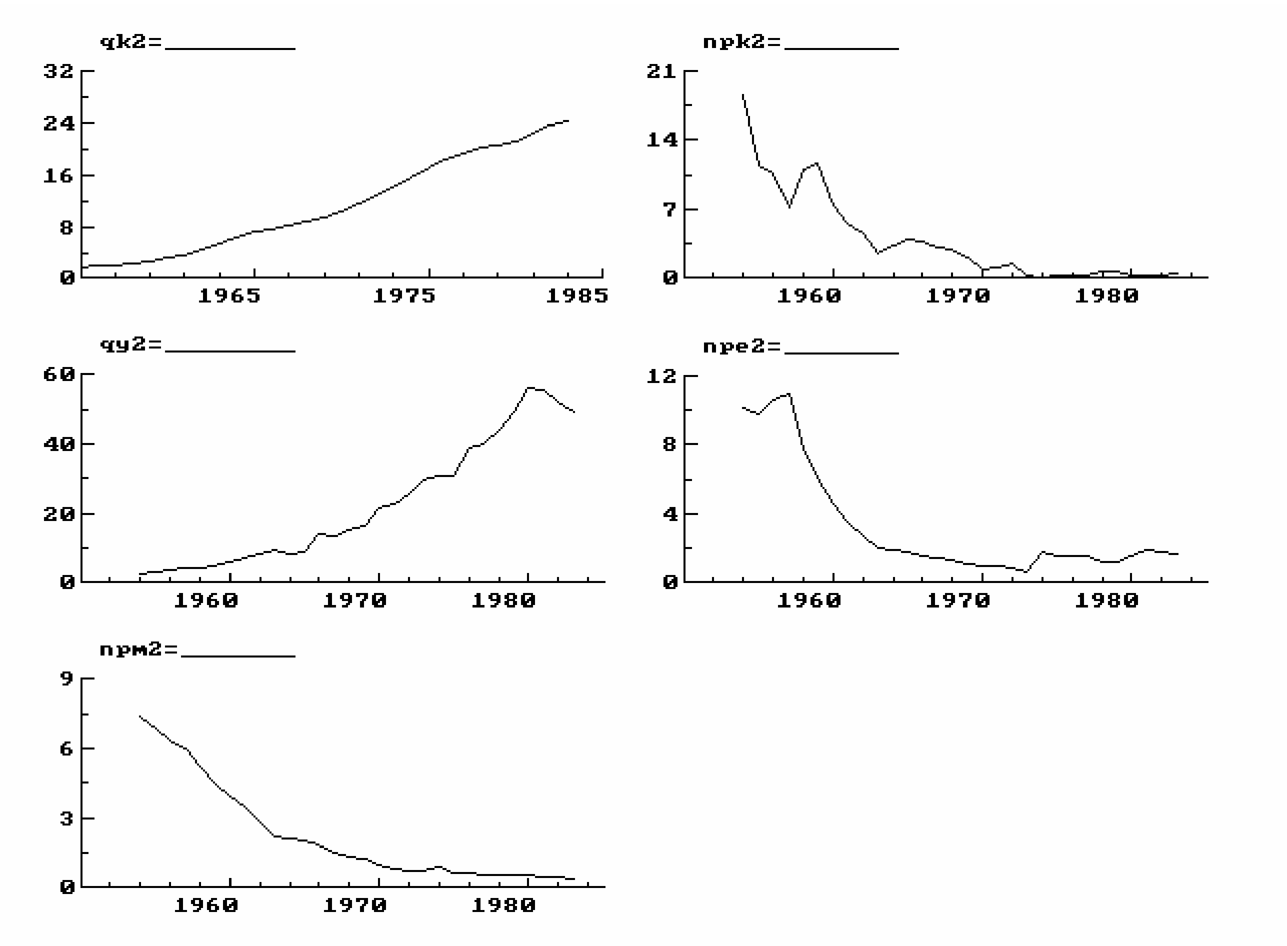

Figure 3. Levels of the squares of capital stock (q^2_k), rental price of capital (np^2_k), output (q^2_y), price of energy (np^2_e) and price of materials (np^2_m)

Figures 1-3 show the plots of the variables and provide a visual tool for assessing their time series properties. What emerges is that almost all the variables are characterized by strong trends, in which case they may be non-stationary. This evidence is confirmed by the results of the augmented Dickey-Fuller (ADF) statistic. Tables 1a-c show that we are not able to reject the null hypothesis of a unit root in favour of stationarity for all nine variables and almost all of their squares. However, the null of a unit root can be rejected for all differenced variables, apart from the levels of labour, capital stock and price of energy. Although the answers provided by the ADF test are ambiguous in only a few cases, we nevertheless checked the correlograms for all variables. The shapes of all correlograms are compatible with non-stationary processes in levels and stationary processes in first-differences. This last result, combined with the previous, non-ambiguous findings from the ADF test, suggests that all variables can be reasonably considered I(1) in conducting the cointegration analysis.

Table 1a. Augmented Dickey-Fuller tests

Series	ADF t-statistic	SE	Lag	t-statistic on lag
q_{ni}	-3.191	0.029	1	2.573
sq_e	-0.305	0.009	2	-1.007
sq_m	-2.427	0.209	1	1.014
sq_l	-0.267	0.018	1	1.638
sq_k	-1.737	0.035	1	3.968
sq_y	-2.015	0.251	2	0.247
np_k	-1.658	0.236	2	-1.319
np_e	-3.186	0.121	2	-1.192
np_m	-2.309	0.049	2	-1.421
q^2_k	-2.458	0.254	1	3.708
np^2_k	-6.245**	0.565	1	0.350
q^2_y	-2.187	2.639	1	1.278
np^2_e	-4.227*	0.265	1	-0.376
np^2_m	-4.013*	0.109	2	-2.115

Notes: ADF t-statistic=t-statistic of the coefficient of the lagged level of the series in the ADF regression with a constant and a linear trend included; SE=standard error of the ADF regression; Lag=order of augmentation in the ADF regression, selected on the basis of the highest significant lag of a four-lag specification; t-statistic on lag=t-statistic of the coefficient of the lagged difference of the series in the ADF regression. The order of this lagged difference is reported in the column Lag; *=rejection of the null at 5% significance level; **=rejection of the null at 1% significance level. Computations obtained using PcGive 8.1 (Doornik and Hendry, 1994a).

Table 1b. Augmented Dickey-Fuller tests

Series	ADF t-statistic	SE	Lag	t-statistic on lag
dq_{ni}	-4.335*	0.032	1	2.082
dsq_e	-4.099*	0.008	1	1.282
dsq_m	-3.632*	0.234	1	0.808
dsq_l	-3.266	0.017	1	1.504
dsq_k	-3.458	0.033	1	2.521
dsq_y	-4.670**	0.265	0	-
dnp_k	-5.925**	0.243	1	2.639
dnp_e	-4.553**	0.142	0	-
dnp_m	-3.982*	0.054	1	1.151
dq^2_k	-2.957	0.268	1	1.842
dq^2_y	-2.378	2.932	1	0.300

Notes: see Table 1a.

Table 1c. Augmented Dickey-Fuller tests

Series	ADF t-statistic	RSE	Lag	t-statistic on lag
d^2sq_l	-3.311	0.023	1	2.430
d^2sq_k	-3.847*	0.045	1	3.955
d^2np_e	-3.240	0.173	1	1.458
$d^2q^2_k$	-3.027	0.373	1	2.851
$d^2q^2_y$	-3.065	3.202	1	2.615

Notes: see Table 1a.

3.2. *SEM estimation*

The model to be estimated is thus formed from the investment equation (14), the energy equation (18), the equation for materials (19) and, finally, the equation for labour (20). As discussed in Section 2.1, additive disturbances have been appended to each equation. Moreover, given the presence of the endogenous net investment in equations (18), (19) and (20), the NL3SLS estimator described in Section 2.1 has been used to estimate the SEM. Valid instruments are given by current, one-period and two-period lagged values of the exogenous variables, as well as two-period and three-period lagged values of the endogenous variables. Finally, first-order residual autocorrelation in the investment and energy equations has been accommodated and the corresponding autocorrelation coefficients have been estimated jointly with the parameters characterizing the firm's technology (see Table 2a). The sufficient condition for identification discussed in Section 2.1 is satisfied. Moreover, the diagnostic tests reported in Table 2b do not suggest any particular problem with the specified model.

Table 2a. SEM estimation

Parameter	Estimate	Standard errors
a_{ku}	0.266**	0.030
a_u	18.672**	3.740
a_{uu}	-3.897**	0.882
a_{uy}	3.131**	0.700
a_{ue}	-0.384	0.158
a_{um}	0.314	0.510
ϕ_{qni}	0.986**	0.047
a_e	-0.128	0.664
a_{ke}	0.171	0.130
a_{ye}	0.172	0.176
a_{ee}	-0.865**	0.257
a_{em}	-0.066	0.218
ϕ_{sqe}	1.044**	0.221
a_m	5.553**	2.162
a_{km}	0.223	0.354
a_{ym}	10.197**	0.614
a_{mm}	-1.581	1.158
a_{mt}	-1.000**	0.224
a_0	18.301*	8.212
a_k	-13.397**	2.740
a_y	-9.234**	3.200
a_t	5.074**	1.009
a_{kk}	5.763*	2.514
a_{ky}	2.021	1.182
a_{kt}	-1.073**	0.215
a_{yy}	0.018	0.426
a_{ut}	-	-
a_{et}	-	-
a_{yt}	-	-

Notes: the present sample is 1957 to 1983. ϕ_{qni}=residual autocorrelation coefficient for the investment equation (14); ϕ_{sqe}=residual autocorrelation coefficient for the energy equation (18); *=rejection of the null at 5% significance level; **=rejection of the null at 1% significance level. Coefficients a_{ut}, a_{et} and a_{yt} have been set equal to zero during estimation. Computations are obtained using Tsp 4.4 (Hall, Cummins and Schnake, 1997) and E-Views (1998).

Table 2b. Diagnostic tests on the estimated SEM

Diagnostics	Investment Equation (14)	Energy equation (18)	Materials equation (19)	Labour equation (20)
R^2	0.836	0.950	0.996	0.507
NORM	1.232 [0.540]	0.442 [0.802]	0.559 [0.756]	1.707 [0.426]
AR(1)	0.997 [0.323]	2.224 [0.136]	0.988 [0.320]	0.401 [0.526]
AR(5)	5.909 [0.315]	5.592 [0.348]	3.427 [0.634]	3.346 [0.647]
ARCH(1)	0.706 [0.401]	0.006 [0.935]	0.784 [0.376]	0.162 [0.687]
ARCH(5)	9.975 [0.076]	2.281 [0.809]	1.288 [0.936]	1.201 [0.945]

Notes: NORM=Jarque-Bera LM test for the null hypothesis of normality of the residuals; AR(1) [AR(5)]=Breusch-Godfrey LM test for the null hypothesis of no residual autocorrelation of order 1 [5]; ARCH(1) [ARCH(5)]=test for the null hypothesis of no residual autoregressive conditional heteroskedasticity of order 1 [5]; P-values are reported in brackets. Computations are made using Tsp 4.4 (Hall, Cummins and Schnake, 1997) and E-Views (1998).

3.3. *VAR estimation*

As discussed in Section 2.2, the starting point for this alternative approach is to model all nine variables forming the KLEM data set (i.e. q_{ni}, sq_e, sq_m, sq_l, sq_y, sq_k, np_k, np_e and np_m) as functions of their own history, a constant term, a linear time trend, an impulse dummy variable for the year 1975 (i1975), and first-differences of the levels of the squares of capital stock, rental price of capital, price of energy, price of materials and output. We must restrict the order of the autoregression to one because of the relatively small sample size compared with the number of estimated equations, although the diagnostics do not suggest this assumption to be unwarranted. The inclusion of the impulse dummy for 1975 is particularly important for the unrestricted VAR to be a statistically adequate model, especially if our concern is normality of residuals and absence of residual autocorrelation (see Table 3). Moreover, it turns out to be a valid way of modeling the lagged effects of the first oil shock, especially for the quantity of net investment (q_{ni}), the quantity of energy (sq_e), the rental price of capital (np_k) and the price of materials (np_m). More specifically, q_{ni} shows three waves or cycles over the analyzed period, with the highest peak occurring in 1974/1975; sq_e attains its maximum in 1974/1975, while np_k its minimum; np_m is characterized by a marked downward-sloping trend over the entire period, with the sole exception of the value recorded by the series in 1975 (see Figures 1-2).

Table 3. Diagnostic tests on the estimated unrestricted VAR

Diagn.	q_{ni}	sq_e	sq_m	sq_l	sq_y	sq_k	np_k	np_e	np_m
SE	0.013	0.003	0.082	0.010	0.058	0.019	0.107	0.058	0.011
RSS	0.002	0.0001	0.074	0.001	0.037	0.004	0.126	0.037	0.001
CAF	0.985	0.999	0.998	0.998	0.999	0.999	0.997	0.998	0.999
AR1	1.850	2.050	0.778	1.010	0.213	1.936	2.874	0.920	0.091
	[0.204]	[0.183]	[0.398]	[0.338]	[0.654]	[0.194]	[0.121]	[0.360]	[0.769]
NORM	1.602	1.827	5.117	0.124	3.075	3.002	0.628	0.070	0.582
	[0.449]	[0.401]	[0.077]	[0.940]	[0.215]	[0.223]	[0.731]	[0.966]	[0.747]
ARCH1	0.082	0.365	0.038	0.239	0.082	0.163	0.715	0.0002	0.664
	[0.780]	[0.561]	[0.849]	[0.636]	[0.780]	[0.696]	[0.419]	[0.989]	[0.436]
		VecPORT4					439.070		
		VecNORM					28.331 [0.057]		

Notes: P-values are reported in brackets; SE=regression standard error; RSS=residual sum of squares; CAF=correlation of actual and fitted values; AR1=single-equation test for the null hypothesis of no residual autocorrelation against first-order autocorrelation, distributed as $F_{(1,10)}$; NORM=single-equation test for the null hypothesis of normality of the residuals, distributed as $\chi^2_{(2)}$; ARCH1=single-equation test for the null hypothesis of no ARCH effects in the residuals against first-order residual autoregressive conditional heteroskedasticity, distributed as $F_{(1,9)}$; VecPORT4=system portmanteau test for the null hypothesis of no residual autocorrelation against fourth-order residual autocorrelation; VecNORM=system test for the null hypothesis of normality of the residuals, distributed as $\chi^2_{(18)}$. Computations are made using PcFiml 8.1 (Doornik and Hendry, 1994b).

The empirical analysis of the cointegration properties of a vector of variables can be conducted by using models (21)-(22) and the Johansen procedure sketched in Section 2.2. Maximum likelihood estimation requires the residuals in (21) to be approximately normal, a condition which seems to be met by the unrestricted VAR. It is well known that the presence of structural breaks and parameter non-constancies leads to an over-estimation of the orders of integration in the univariate framework. This phenomenon is likely to persist in a multivariate context and can affect, or even impede, the determination of the number of valid cointegrating vectors. Moreover, it is important to use the appropriate critical values for the cointegration likelihood-ratio test statistics, since their asymptotic distribution is not χ^2, but rather a generalization of the Dickey-Fuller distribution, with a structure depending on the nature of the problem. In the cointegration analysis, we have restricted the constant term and the linear trend to enter the cointegrating space, with no restrictions on the dummy variable and the first differences of the second-order terms appearing in the cost function.

Table 4. Cointegration analysis of the UVAR

Null hypothesis	Johansen's trace test	95% critical values
$\rho = 0$	471.50	156.44
$\rho \leq 1$	331.00	130.90
$\rho \leq 2$	230.80	108.03
$\rho \leq 3$	159.90	85.35
$\rho \leq 4$	95.07	65.48
$\rho \leq 5$	57.58	45.76
$\rho \leq 6$	24.42	30.72
$\rho \leq 7$	7.40	16.71
$\rho \leq 8$	2.10	-

Notes: ρ is the rank of the m×m long-run matrix Π in model (3). The appropriate critical values for the Johansen trace test have been computed using the simulation program DisCo developed by Johansen and Nielsen (1993). The cointegration analysis has been conducted using PcFiml 8.1 (Doornik and Hendry, 1994b). It is not possible to obtain the critical values of the trace test under the null that $\rho \leq 8$, because m-ρ should be at least equal to the number of restricted drift terms (see Johansen and Nielsen, 1993).

The Johansen trace test suggests the presence of five LR stationary relationships (see Table 4). This result means that we can rewrite the m×m LR matrix Π in (22) as $\Pi = \alpha_{m\times\rho}\beta'_{\rho\times m}$, where m=11 (i.e. 9 endogenous variables plus constant and linear trend) and ρ=5. In Section 2.2 it was noted that α and β' are not unique. In fact, given any non-singular $\rho\times\rho$ matrix ζ, we can define $\alpha^*=\alpha\zeta^{-1}$ and $\beta^*=\beta\zeta$, such that $\Pi=\alpha^*\beta^{*\prime}=\alpha\beta'$. The important implication is that it is possible to choose ζ such that β^* has an economic interpretation and test the set of restrictions which the selected ζ imposes on the unrestricted cointegrating vectors.

The theory of dynamic duality permits an economic interpretation of the five LR equilibria, namely the optimal LR levels of investment, energy, materials, labour and capital stock expressed by equations (14), (18), (19), (20) and (17), respectively. The corresponding set of restrictions on the cointegrating vectors is not rejected by the data, being the computed value of the likelihood-ratio test statistic $\chi^2_{(3)}$=3.480 with a P-value of 0.323. The estimated restricted cointegrating vectors are reported in Table 5.

Table 5. Restricted estimates of $\beta^{*\prime}$ eigenvectors

	q_{ni}	sq_e	sq_m	sq_l	sq_k	sq_y	np_k	np_e	np_m	Const.	Trend
$\beta^{*\prime}_1$	-1.0	0.0	0.0	0.0	0.0	-0.163**	-0.044**	0.110**	-0.417**	0.961**	0.000
						(0.02)	(0.01)	(0.02)	(0.05)	(0.09)	
$\beta^{*\prime}_2$	0.0	-1.0	0.0	0.0	0.0	-0.117**	-0.003	0.096**	-0.383**	0.886**	0.000
						(0.01)	(0.01)	(0.01)	(0.03)	(0.05)	
$\beta^{*\prime}_3$	0.0	0.0	-1.0	0.0	0.0	2.073**	0.168	0.482	0.182	-4.919**	-0.063**
						(0.24)	(0.15)	(0.27)	(0.73)	(1.30)	(0.01)
$\beta^{*\prime}_4$	0.0	0.0	0.0	-1.0	0.0	0.432**	0.015	0.021	-0.006	0.490**	-0.047**
						(0.05)	(0.02)	(0.03)	(0.08)	(0.15)	(0.01)
$\beta^{*\prime}_5$	0.0	0.0	0.0	0.0	-1.0	2.110**	0.195	1.249**	-1.557	-3.187**	0.000
						(0.26)	(0.17)	(0.31)	(0.84)	(1.50)	

Notes: $\beta^{*\prime}_j$ indicates the j-th row of the 5×(9+2) $\beta^{*\prime}$ matrix, j=1,...,5. Standard errors are reported in parentheses; **=rejection of the null hypothesis at the 1% significance level.

The next stage of the VAR methodology is to seek for a more parsimonious representation of the original system. This requires a test for weak exogeneity of a subset of variables within the VAR, as illustrated in Section 2.2. It is important to bear in mind that the traditional literature on factor demands generally assumes output and factor prices to be exogenous in the equations specifying the optimal quantities of production inputs used by the firm. Thus, it seems natural to exploit the VAR estimation and cointegration analysis in order to test whether output and factor prices can be considered as valid conditioning variables. If the null hypothesis of weak exogeneity of output and factor prices is not rejected, the nine-dimensional VAR reduces to a five-dimensional system, where prices and the level of production are valid non-stochastic regressors.

The condition for weak exogeneity of output and factor prices is that none of the five cointegrating vectors is significant in the equations for sq_y, np_k, np_e and np_m of model (22). A formal test of weak exogeneity requires the estimation of model (22) (see Table 6).

Table 6. VAR-ECM estimation

Variab.	dq_{ni}	dsq_e	dsq_m	dsq_l	dsq_y	dsq_k	dnp_k	dnp_e	dnp_m
$rvec_{1,t-1}$	-0.604**	-0.156**	-3.014*	-0.288	-3.156**	-0.490	3.988*	-0.511	0.298*
	(0.171)	(0.037)	(1.120)	(0.152)	(0.857)	(0.253)	(1.409)	(0.571)	(0.134)
$rvec_{2,t-1}$	0.632	-0.435**	-5.233	0.651	-4.715*	-0.620	2.885	0.545	0.281
	(0.389)	(0.085)	(2.540)	(0.344)	(1.944)	(0.574)	(3.195)	(1.295)	(0.304)
$rvec_{3,t-1}$	-0.122	-0.004	-1.875**	-0.017	-1.283**	-0.253*	1.692*	-0.702**	-0.099
	(0.071)	(0.015)	(0.463)	(0.063)	(0.354)	(0.105)	(0.582)	(0.236)	(0.055)
$rvec_{4,t-1}$	-0.129	-0.142*	0.476	-0.155	-1.318	-0.170	4.332*	-1.153	-0.292
	(0.242)	(0.053)	(1.583)	(0.214)	(1.212)	(0.358)	(1.991)	(0.807)	(0.189)
$rvec_{5,t-1}$	0.152	-0.014	1.468*	0.049	1.173**	0.319*	-2.039**	0.704*	0.088
	(0.076)	(0.016)	(0.496)	(0.067)	(0.380)	(0.112)	(0.625)	(0.253)	(0.059)
Trend	0.001	-0.0000	-0.013	-0.0008	-0.003	0.003	-0.030	0.007	0.001
	(0.001)	(0.0004)	(0.012)	(0.001)	(0.009)	(0.003)	(0.015)	(0.006)	(0.001)
Const.	-0.044	0.003	0.244	-0.002	-0.027	-0.089	1.015*	-0.219	-0.029
	(0.051)	(0.011)	(0.334)	(0.045)	(0.256)	(0.076)	(0.420)	(0.170)	(0.040)
dq^2_k	0.032	-0.001	0.255	-0.031	0.291**	0.154**	-0.577**	0.172	0.011
	(0.019)	(0.004)	(0.121)	(0.016)	(0.093)	(0.027)	(0.153)	(0.062)	(0.014)
dnp^2_k	-0.010**	-0.001*	-0.025	-0.006*	-0.039**	-0.008*	0.199**	0.019*	0.001
	(0.002)	(0.0005)	(0.017)	(0.002)	(0.012)	(0.004)	(0.020)	(0.008)	(0.002)
dnp^2_e	-0.020	0.002	-0.004	-0.008	-0.063	-0.021	0.178	0.139**	-0.034
	(0.010)	(0.002)	(0.066)	(0.009)	(0.051)	(0.015)	(0.083)	(0.034)	(0.008)
dnp^2_m	-0.030	-0.032**	-0.371	0.004	-0.259	-0.032	-0.083	0.199	0.403**
	(0.033)	(0.007)	(0.217)	(0.029)	(0.166)	(0.049)	(0.273)	(0.111)	(0.026)
dq^2_y	0.006**	0.001**	0.079**	0.003	0.099**	0.009**	-0.047**	0.009	0.001
	(0.002)	(0.0004)	(0.012)	(0.002)	(0.009)	(0.003)	(0.015)	(0.006)	(0.001)
i1975	-0.072**	0.014**	-0.016	-0.003	-0.091	-0.052	0.458*	-0.161*	-0.080**
	(0.020)	(0.004)	(0.128)	(0.017)	(0.098)	(0.029)	(0.161)	(0.065)	(0.015)

F-tests	$rvec_{1,t-1}$	$rvec_{2,t-1}$	$rvec_{3,t-1}$	$rvec_{4,t-1}$	$rvec_{5,t-1}$
$F_{(9,6)}$	12.831	16.015	17.436	3.814	14.416
	[0.003]	[0.001]	[0.001]	[0.059]	[0.002]

Notes: $rvec_j$ indicates the j-th restricted cointegrating vector, j=1,...,5; standard errors are in parentheses, P-values are in brackets; F-tests are for the joint significance of each restricted cointegrating vector in all VAR-ECM equations; **(*)=rejection of the null hypothesis at the 1% (5%) significance level.

From Table 6, we notice that each cointegrating vector is significant in at least one equation. If we impose the restrictions for weak exogeneity on model (22) (the number of these restrictions is given by the number of equations times the number of cointegrating vectors, i.e. 4x5=20) and compare the unrestricted and restricted log-likelihoods with a likelihood-ratio test, we obtain $\chi^2_{(20)}$ = 416.25, which strongly rejects the restrictions and suggests that output and factor prices are endogenous.

In Section 2.2 the notion of model encompassing has been briefly discussed. Whereas a direct comparison between the SEM and VAR specifications is not feasible because of the endogeneity of output and input prices, it is however possible to compare the estimated SEM with its corresponding URF (see equations 24 and 25). The URF corresponds to the VAR model once the exogeneity of output and input prices is imposed. The URF can be obtained in the following way. First, substitute the investment equation (14) into the energy equation (18), the materials equation (19) and the labour equation (20). Second, substitute both equations for energy and materials into the labour equation. Finally, include one-period lags for all dependent variables among the regressors. The URF is then composed by one equation for each of the four endogenous variables q_{ni}, sq_e, sq_m and sq_l. The regressors of the first three equations are a constant term, a linear time trend, sq_k, np_k, np_e, np_m, sq_y, and the lagged value of the dependent variable. The labour equation has, as additional regressors, the squares and the cross products among sq_k, np_k, np_e, np_m, sq_y and the time trend (20 additional regressors, since the quadratic time trend term is not included in the specification, given the absence of a non-linear trend in the variables). The URF can be consistently estimated with Ordinary Least Squares and it is statistically adequate when tested against first-order autocorrelation, non-normality and first-order conditional heteroskedasticity of the residuals. The estimated SEM can now be interpreted as a set of testable restrictions imposed on the URF. By simply comparing the SEM with the URF, the number of these restrictions given by the number of squares and cross products plus the number of lagged dependent variables (i.e. 20+4=24). If the restrictions imposed on the URF by the SEM are not rejected by the data, the SEM is said to parsimoniously encompassing the URF (see, e.g., Clements and Mizon, 1991). Parsimonious encompassing is usually tested via a likelihood-ratio test, which, in the present context, is $\chi^2_{(24)}$ = 152.36. The strong evidence against the SEM should be interpreted with some care. In particular, it is important to remember that the SEM is statistically inadequate only when it is contrasted with its corresponding URF (see diagnostics reported in Table 2b). Moreover, since exogeneity of output and input prices has been strongly rejected by the data, the URF, although statistically adequate, can hardly be considered a “good” model and the failure of the SEM to encompass the URF does not imply that the URF is “better” (see, among others, Mizon, 1984, p. 289).

4. Practical use of SEM and VAR models of factor demands

4.1. *Estimating price elasticities and adjustment costs*

In this section we show how to use both the SEM and VAR models to calculate some economic indicators which are widely used to describe the production structure of a manufacturing sector.

From the SEM, the elasticity of the n-th factor demand to the ν-th input price can be computed according to the following expressions:

$$(26)\quad e_{n\nu}^{SR} = \left.\frac{\partial s\hat{q}_n}{\partial np_\nu}\right|_{sq_k\ fixed}$$

$$e_{n\nu}^{IR} = \left.\frac{\partial s\hat{q}_n}{\partial np_\nu}\right|_{sq_k=\lambda sq_k^*}$$

$$e_{n\nu}^{LR} = \left.\frac{\partial s\hat{q}_n}{\partial np_\nu}\right|_{sq_k=sq_k^*}$$

where all variables are log transformed, the long-run level sq^*_k is given by (17) and the superscript ^ indicates fitted values. The distinction between short-run (SR), intermediate-run (IR) and long-run (LR) elasticities in empirical factor demand analysis goes back to Berndt, Fuss and Waverman (1980). A SR elasticity assumes that the quasi-fixed factor sq_k is fixed, whereas a LR elasticity is calculated by evaluating sq_k at its optimal long-run level sq^*_k. An IR elasticity is based on a proportion λ of the complete adjustment sq^*_k: since the data frequency is annual, the term λsq^*_k captures the adjustment of sq_k towards sq^*_k after one year.

From the VAR-ECM, it is straightforward to obtain the price elasticities as follows:

$$(27)\quad e_{n\nu}^{SR} = \frac{\partial d\hat{s}q_n}{\partial np_\nu}$$

$$e_{n\nu}^{IR} = e_{n\nu}^{SR} + \sum_{j=1}^{\rho}\frac{\partial \hat{s}q_n}{\partial r\hat{v}ec_j}\cdot\frac{\partial r\hat{v}ec_j}{\partial np_\nu}$$

$$e_{n\nu}^{LR} = e_{n\nu}^{SR} + \sum_{j=1}^{\rho}\frac{\partial r\hat{v}ec_j}{\partial np_\nu}$$

where j indicates the j-th restricted cointegrating vector and ρ=5. As for elasticities (26), the discriminant between SR, IR and LR elasticities is given by the degree of adjustment towards the long-run. However, the main differences with elasticities (26) are that the equilibrium is not unique (actually, the number of equilibrium relationships is five) and that the system as a whole (not only sq_k) adjusts towards the long-run. Naturally, the peculiarities of the VAR reflect on the way the factor demand elasticities (27) are computed. In particular, the term $\sum_{j=1}^{\rho} \frac{\partial \hat{s}q_n}{\partial r\hat{v}ec_j} \cdot \frac{\partial r\hat{v}ec_j}{\partial np_\nu}$, which indicates partial adjustment towards each of the five equilibria, is composed by two parts: i) $\partial \hat{s}q_n / \partial r\hat{v}ec_j$, the loading coefficient in the VAR corresponding to the λ coefficient in the SEM; ii) $\partial r\hat{v}ec_j / \partial np_\nu$, the LR elasticity of the n-th input demand to the ν-th input price. It is worth noticing that in the LR elasticity formula only part ii) is present.

A closer examination of the SEM and VAR elasticities can help the applied researcher to better interpret the empirical results and can be used as a basis to discriminate between the two approaches. For example, if we concentrate on variable inputs, we can denote energy and materials with the indexes n and ν, respectively. Remembering that the price of labour (p_l) has been chosen in the quadratic specification as the numeraire, i.e. $np_\nu \equiv p_\nu / p_l$, the SEM elasticities (26) can be written as:

$$(28)\quad e_{n\nu}^{SR} = \frac{r \cdot \hat{a}_{n\nu}}{\bar{p}_l} - \frac{r \cdot \hat{a}_{kn}\hat{a}_{ku}\hat{a}_{u\nu}}{\bar{p}_l}$$

$$e_{n\nu}^{LR} = \frac{r \cdot \hat{a}_{n\nu}}{\bar{p}_l} + \frac{1}{\hat{a}_{ku} - r} \cdot \frac{r \cdot \hat{a}_{kn}\hat{a}_{ku}\hat{a}_{u\nu}}{\bar{p}_l}$$

$$e_{n\nu}^{IR} = \frac{r \cdot \hat{a}_{n\nu}}{\bar{p}_l} + r \cdot \hat{\lambda} \cdot \hat{a}_{kn} \left(\frac{r \cdot \hat{a}_{ku}}{\hat{a}_{ku} - r} \cdot \frac{\hat{a}_{u\nu}}{\bar{p}_l} \right),$$

where $\bar{p}_l$ indicates the sample mean of p_l.

Recalling that the estimated equation for the n-th variable input in the VAR-ECM representation is given by:

$$ds\hat{q}_{nt} = \hat{\alpha}_{n1} r\hat{v}ec_{1,t-1} + \hat{\alpha}_{n2} r\hat{v}ec_{2,t-1} + \ldots + \hat{\alpha}_{n5} r\hat{v}ec_{5,t-1} + \hat{\mu}_1 t + \hat{\mu}_0 + \hat{\gamma}_k dq_{kt}^2 +$$

$$+\hat{\gamma}_u dnp_{kt}^2 + \hat{\gamma}_n dnp_{nt}^2 + \hat{\gamma}_\nu dnp_{\nu t}^2 + \gamma_y dq_y^2 + \hat{\gamma}_d \mathrm{i}1975_t \,,$$

whereas the n-th restricted cointegrating vector is:

$$r\hat{v}ec_{i,t-1} = sq_{n,t-1} - \hat{\beta}_{ny} sq_y - \hat{\beta}_{nk} np_k - \hat{\beta}_{nn} np_n - \hat{\beta}_{n\nu} np_\nu - \hat{c}_0 - \hat{c}_1 t \,,$$

then the VAR elasticities (27) take the following expressions:

$$(29)\quad e_{n\nu}^{SR} = 2\hat{\gamma}_\nu \frac{\overline{p}_\nu}{\overline{p}_l^2}$$

$$e_{n\nu}^{IR} = e_{n\nu}^{SR} + \frac{1}{\overline{p}_l} \sum_{j=1}^{\rho} \hat{\alpha}_{nj} \hat{\beta}_{j\nu}$$

$$e_{n\nu}^{LR} = e_{n\nu}^{SR} + \frac{1}{\overline{p}_l} \sum_{j=1}^{\rho} \hat{\beta}_{j\nu} \,.$$

If we draw our attention to the LR, it is interesting to note that both SEM and VAR elasticities of the demand for input n to the price of input ν are formed by two components. The first one is the corresponding SR elasticity, whereas the second reflects the system LR equilibrium. In the SEM specification, this second component depends on the estimated parameters which characterize the LR level of the quasi-fixed factor capital (see equation 17). Conversely, in the VAR model, the estimated cointegrating coefficients with which the price of input j enters each of the 5 cointegrating vectors appear in the second component of the LR elasticity (see Table 5). The motivation of this asymmetry is evident. The cointegrated VAR is designed to model the presence of multiple equilibria in the system of factor demands, whereas the SEM model assumes that the system depends only on the equilibrium level of the quasi-fixed factor. Within the VAR, each input simultaneously adjusts to its LR equilibrium; in the SEM, the adjustment of each input to its LR level follows the adjustment of the quasi-fixed factor. This different way of modeling the LR equilibrium has important consequences on the estimated LR elasticities, which are expected to be (on average) larger using the VAR approach.

This conclusion cannot be extended to IR elasticities. From expressions (28) and (29), it is easy to see that a third component plays a crucial role, namely the speed of adjustment towards the LR equilibrium. In the SEM model, this component is unique (see equation 16), while in the VAR specification it is represented by the coefficients (loadings) with which the 5 cointegrating vectors enter each input demand equation (see Table 6).

It is also interesting to compare the SEM and VAR models on the basis of their ability to produce credible measures of ADC. For both models, total ADC (TADC) have been computed by summation over all terms depending on net investment sq^*_{ni} in the dual cost function:

(30) $\hat{C}(\cdot) = s\hat{q}_l^* + np_e s\hat{q}_e^* + np_m s\hat{q}_m^*$.

The difference between the two approaches lies in the specification of the elements sq^*_l, sq^*_e and sq^*_m in (30). For the SEM, these are given by equations (20), (18) and (19), respectively. For the VAR, they are derived from the corresponding equations in the VAR-ECM system (see Table 6). Given the identifying restrictions on the cointegrating vectors discussed in Section 3, the only non-zero coefficient on net investment appears in the first restricted cointegrating vector (see first line of Table 6). The main implication should be a non-negligible under-estimation of the ADC component with the VAR approach. Again, it is the different way of modeling adjustment costs followed by the two approaches that motivates this discrepancy. The SEM model adopted in this paper incorporates ADC into the firm's dynamic optimization problem through an explicit function of the amount of investment in the quasi-fixed input (equations 14 and 15), which directly enters the demand equation for variable inputs. Conversely, the way the VAR approach takes into account the presence of ADC is only implicit, and this is realized by treating investment as an additional factor demand equation. When we use (30) to estimate the ADC component of total costs, the contribution of the VAR model is limited to the LR effect of net investment on the demand for variable inputs (first restricted cointegrating vector in the VAR-ECM representation reported in Table 6), whereas the SEM approach considers both the direct effect of the adjustment of the quasi-fixed factor towards its LR level and the indirect effects of this adjustment on the variable inputs.

Some point estimates of price and output elasticities from the SEM and the VAR are reported on the top of Tables 7 and 8, respectively. At first inspection, all the elasticities are very low, whereas, as expected, the LR elasticities calculated on the basis of the VAR model are systematically higher than those estimated using the SEM coefficients.

SR, IR and LR direct price elasticities calculated on the basis of the SEM model are always negative as suggested by economic theory. LR elasticities of net investment with respect to input prices and output quantity are zero by construction, since the stock of capital has completed its adjustment to the desired level in the LR. SR, IR and LR investment, energy and materials elasticities with respect to output are positive, as predicted by economic theory. When the VAR is considered, only the SR and IR direct price elasticities for energy, the LR direct price elasticity and the IR output elasticity of materials are characterized by incorrect signs. The percentage of VAR elasticities which satisfy the Le Chatelier principle is 80%. This percentage drops to 53% if the SEM elasticities are considered. The relatively poor performance of the SEM can be rationalized in terms of the more rigid representation of the LR equilibrium associated with the SEM (i.e. a unique cointegrating vector), as well as the role played by the LR equilibrium in expressions (28) and (29).

The cross-price elasticities calculated from the SEM model are all below one, whereas half of the LR elasticities based on the VAR estimates are close to or even greater than two. From the sign of the cross-price elasticities it is possible to obtain information about factor substitution and complementarity. If the SEM model is considered, substitution relationships emerge between capital and materials, capital and labour, energy and labour, and materials and labour, whereas capital and energy, and energy and materials appear to be complementary factors of production. The situation does not change significantly when the VAR estimates are used: in this case, the only difference is given by the relationship between capital and materials, which are now viewed as complementary inputs.

A few more specific considerations can help to characterize the evolution of the Italian total manufacturing sector over the sample period. The elasticity of investment demand with respect to the price of labour is positive when measured using the SEM estimates, negative in the IR, but positive in the SR and LR when estimated on the basis of the VAR. This evidence suggests that capital accumulation is obtained through labour substitution.

Table 7. SEM: price and output elasticities, ADC indicators (mean values, 1957-1983)

Elasticities	Short-run (SR)	Intermediate-run (IR)	Long-run (LR)
e_{ipk}	-0.098 (0.079)	-0.080 (0.064)	-
e_{iqy}	0.064 (0.000)	0.052 (0.000)	-
e_{ipe}	-0.010 (0.008)	-0.008 (0.006)	-
e_{ipm}	0.008 (0.006)	0.006 (0.005)	-
e_{ipl}	0.211 (0.259)	0.171 (0.210)	-
e_{epk}	-0.020 (0.016)	-0.038 (0.030)	-0.043 (0.035)
e_{eqy}	0.002 (0.000)	0.014 (0.000)	0.018 (0.000)
e_{epe}	-0.080 (0.065)	-0.082 (0.066)	-0.083 (0.067)
e_{epm}	-0.008 (0.006)	-0.006 (0.005)	-0.006 (0.005)
e_{epl}	0.199 (0.244)	0.238 (0.291)	0.250 (0.305)
e_{mpk}	0.052 (0.042)	0.028 (0.023)	0.021 (0.017)
e_{mqy}	0.774 (0.000)	0.789 (0.000)	0.794 (0.000)
e_{mpe}	-0.004 (0.003)	-0.006 (0.005)	-0.007 (0.006)
e_{mpm}	-0.152 (0.122)	-0.149 (0.121)	-0.149 (0.120)
e_{mpl}	0.145 (0.163)	0.196 (0.223)	0.212 (0.242)
Total costs (TC)		4.240 (0.311)	
Total ADC (TADC)		0.904 (0.950)	
TADC/TC		0.213 (0.228)	
TADC/qy		0.217 (0.192)	
TADC/pk		1.237 (2.638)	

Notes: $e_{n\nu}$=price elasticities, n=investment (i), energy (e), materials (m), ν=price of capital (pk), price of energy (pe), price of materials (pm), price of labour (pl); e_{nqy}=output elasticities. Standard deviations are given in brackets.

The elasticity of energy demand to the price of labour is always positive, regardless of the time period (SR, IR or LR) or the model (SEM or VAR). Over the estimated period, the production structure operates by substituting the more expensive input (energy) with labour. Conversely, given the negative sign of the elasticity of energy demand with respect to the rental price of capital, a rise in the price of energy seems to cause a reduction in the capital stock.