What is behind ambiguity aversion? An experiment disentangling model uncertainty and risk aversion

Loïc Berger\textsuperscript{12} Valentina Bosetti\textsuperscript{13}

\textsuperscript{1}Fondazione Eni Enrico Mattei (FEEM) and
\textsuperscript{2}Federal Planning Bureau (FPB)
\textsuperscript{3}Department of Economics, Bocconi University

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Motivation (1)

Uncertainty is ubiquitous.

Deep uncertainty/ambiguity is more and more recognized to play a central role in decision making processes

- Partly because of some recent "catastrophic" events
  - Economic uncertainty: financial crisis
  - Technological uncertainty: Fukushima

- Partly because of a growing awareness about
  - Environmental uncertainty: climate change
  - Demographic uncertainty: longevity / mortality risk
What is ambiguity/deep uncertainty/Knightian uncertainty?

→ "all kind of situations in which a decision maker does not have sufficient information to quantify through a single probability distribution the stochastic nature of the problem she is facing"

(Cerreia-Vioglio et al., 2013)
Motivation (3)

All uncertainty relevant for decision making is ultimately subjective

- Yet, in applications (especially with data) it is convenient to distinguish between *aleatory* and *epistemic* uncertainty

*Aleatory uncertainty* (=risk):

- Examples: games of chance (roulette, coin, dice), measurement errors
- Deals with variability in data (because of inherent randomness, measurement errors, omitted minor explanatory variables)
- Characterizes data generating processes (DGP) (i.e. probability models)
- Probability is a measure of randomness/variability

*Epistemic uncertainty* (=model uncertainty):

- Deals with the truth of propositions: "the composition of the urn is 50 red and 50 black balls" "the parameter that characterizes the DGP has value x"
- Probability is a measure of degree of belief

⇒ Deep uncertainty/ambiguity: combination of both aleatory and epistemic uncertainty
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Introduction

Research questions

From Ellsberg (1961), we know that people are generally ambiguity averse but...

1. What is the source of ambiguity non neutrality? (normative vs. descriptive)
2. What is the degree of ambiguity aversion?
   ⇒ What is the degree of model uncertainty aversion (in comparison to risk aversion)?
3. What are the underlying properties of the ambiguity aversion function? (CAAA?, DAAA?)
Why is it important for us?

Deep uncertainty plays an important role in the economic and policy science related to climate change.
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These uncertainties arise from:

- the underlying science of climate (and the extreme complexity of the climate system)
- our inability to perfectly capture the way our socio-economic system would respond, mitigate and adapt to climate change
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⇒ in many situations, the probabilistic model is neither explicitly given, nor can be perfectly approximated or inferred with the available data and current scientific methods
Examples (1)

Heal and Millner (2013)  
IPCC (2013)
Introduction

Examples (2)

Probability of climate catastrophe? (Berger, Emmerling, Tavoni; 2015)
Roadmap

1. Experimental procedures
2. Theoretical predictions
3. Results
4. Conclusion
1. Experimental procedure
The choices situations

**Urn 1:**
*(risk)*

- 50 black balls
- 50 red balls

The number of red and black balls is perfectly known.
1. Experimental procedure

The choices situations

Urn 2:
(compound risk)

The number of red and black balls is determined by flipping a fair coin in the air.

100 black

100 red
The choices situations

Urn 3: (model uncertainty)

The number of red, black and the total number of balls in the urn are unknown, but information is provided by two "experts", each giving her own assessment of the composition of the urn.

- Expert 1 says there are only Red balls.
- Expert 2 says there are only Black balls.

If your bet is correct, you win 15 euros. If your bet is incorrect, nothing happens.
The choices situations

Urn 4:  
(Ambiguity à la Ellsberg)

The total number of balls is given, but the exact composition of the urn is unknown
# The choices situations

<table>
<thead>
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<td>unknown</td>
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<td>compound</td>
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Random lottery pairs (RLP)

- Urns 1, 2, 3 and 4 presented 2-by-2
- Randomized sequence
- In each decision, subjects have to:
  1. place a bet on the color (Red or Black) drawn
  2. decide which of the two urns presented to place their bet (or whether indifferent)

If bet is correct: win €15, otherwise nothing happens
In general:

- Urn 1 ≽ Urn 4: Ellsberg (1961),... (Trautmann and van Kuilen, 2014) → Ambiguity aversion
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In general:

- **Urn 1 ≽ Urn 4:** Ellsberg (1961),... (Trautmann and van Kuilen, 2014)
  → Ambiguity aversion

  → Compound lottery aversion

⇒ Attitudes to ambiguity and compound lottery are tightly associated
2. Theoretical predictions
Two layers uncertainty (Cerreia-Vioglio et al., 2013; Marinacci, 2015)

- Distinction between risk (aleatory uncertainty) and model uncertainty (epistemic uncertainty)
- Possible alternative models $P_\theta(r)$ (i.e. compositions of urns) belong to a set $M$ taken as given
- Single prior probability measure over models: $q_\theta$
- The DM chooses the act that maximizes his utility:

$$U(f_i) = E_\theta(v \circ u^{-1}) \left( \sum_{c \in \{r_i, b_i\}} \tilde{P}_\theta(c) u(f_i(c)) \right), \ i \in \{1, \ldots, 4\}$$
A theory of choice under uncertainty

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In particular:

- **Risk (Urn 1):** $M = \{ P(r) = 1/2 \}$ is a singleton

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- **Compound risk (Urn 2):** \( M = \{ P(r) = 1, P(r) = 0 \} \)
  
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  \( (2) \)
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- **Model uncertainty (Urn 3):** $M = \{P(r) = 1, P(r) = 0\}$
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  U^d(f_3) = \sum_{c \in \{r_3, b_3\}} \frac{1}{2} v(f_3(c)).
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- **Ellsberg** (Urn 4): \( M = \{ P_\theta(r) : P(r) = \frac{\theta - 1}{N} \text{ for } \theta = \{1, \ldots, 101\} \} \)

  \[ U(f_4) = \frac{1}{101} \sum_{\theta=1}^{101} (v \circ u^{-1}) \sum_{c \in \{r_4, b_4\}} P_\theta(c) u(f_4(c)) . \quad (4) \]
Predictions

Let $C_i$ be the CE for urn $i$, and $C_0$ be the sure amount that corresponds to the expected gain obtained in the uncertain situation, then:

**H1:** $C_0 \geq C_1$ and $C_0 \geq C_3$ Risk and model uncertainty aversion
2. Theoretical predictions

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**H2:** $C_1 = C_2 \geq C_3 \iff C_1 \geq C_4$ (i.e. $-\frac{v''}{v'} \geq -\frac{u''}{u'} \iff -\frac{\phi''}{\phi'} \geq 0$)

Reduction of compound risk, stronger aversion to model uncertainty than to risk, ambiguity aversion
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**H3:**

\[
\frac{\partial}{\partial w} \left[ -\frac{u''(w)}{u'(w)} \right] \leq 0 \quad \text{and} \quad \frac{\partial}{\partial w} \left[ -\frac{u''(w)}{u'(w)} w \right] \geq 0
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Decreasing absolute risk aversion (DARA)
Increasing relative risk aversion (IRRA)

**H4:**

\[
\frac{\partial}{\partial U} \left[ -\frac{\phi''(U)}{\phi'(U)} \right] \leq 0
\]

Decreasing absolute ambiguity aversion (DAAA)
3. General results
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Implementation

- 189 subjects (Bocconi students)
- 12 sessions of +/- 75 min (instructions, training session, 9 tasks, questionnaire, payment)
- 9 tasks: 1 RLP task, 2 CE tasks, 6 MPL tasks
- Random incentive system
- Average gain: €18.5 (min €5, max €40, median €9)
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- Robustness round: 91 subjects at COP21 (negotiators, NGOs, ...)

3. General results
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RQ1: What is the source of ambiguity non neutrality?

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Note: Chi-square test: 0.085 Chi-square test: 8.9e-09
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- 79% (150/189) are ambiguity non neutral
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- Stronger association between ambiguity and reduction of compound uncertainty
RQ1: What is the source of ambiguity non neutrality?

Logistic regressions:

- Dependent variable: Ambiguity neutrality ($C_1 = C_4$)
- Regressor:
  - reduction of compound risk ($C_1 = C_2$)
  - reduction of compound uncertainty ($C_1 = C_3$)

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Notes: 189 observations, *** $p$ – value < 0.001
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- Probability of \(C_1 = C_4\) is 21%
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Notes: 189 observations, *** \(p - value < 0.001\)

- Probability of \(C_1 = C_4\) is 21%
- it increases to 46% if \(C_1 = C_3\), and decreases to 9% if \(C_1 \neq C_3\)
RQ1: What is the source of ambiguity non neutrality?

Logistic regressions:

- Dependent variable: Ambiguity neutrality \((C_1 = C_4)\)
- Regressor:
  - reduction of compound risk \((C_1 = C_2)\)
  - reduction of compound uncertainty \((C_1 = C_3)\)

<table>
<thead>
<tr>
<th></th>
<th>Odds Ratio</th>
<th>Standard Error</th>
<th>Lower 95% Confidence Interval</th>
<th>Upper 95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1 = C_2)</td>
<td>2.151</td>
<td>0.973</td>
<td>0.886</td>
<td>5.222</td>
</tr>
<tr>
<td>(C_1 = C_3)</td>
<td>8.297***</td>
<td>3.320</td>
<td>3.787</td>
<td>18.176</td>
</tr>
</tbody>
</table>

Notes: 189 observations, *** \(p - value < 0.001\)

- Probability of \(C_1 = C_4\) is 21%
- it increases to 46% if \(C_1 = C_3\), and decreases to 9% if \(C_1 \neq C_3\)
- it increases to 13% if \(C_1 = C_2\), and decreases to 2% if \(C_1 \neq C_2\)
RQ1: What is the source of ambiguity non neutrality?

Result 1:
Attitudes to ambiguity and compound uncertainty are closely related (association much stronger when subjective probabilities than when objective ones)
RQ2: What is the degree of ambiguity aversion?

(a) How do risk and model uncertainty aversion differ?
   - 2 certainty equivalent (CE) tasks: risk and model uncertainty
   - Sequence of 10 decisions

<table>
<thead>
<tr>
<th>Decision</th>
<th>Option A</th>
<th>Option A</th>
<th>Option B</th>
<th>Option B</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$P(\text{Red})$</td>
<td>outcome</td>
<td>$P(\text{Black})$</td>
<td>outcome</td>
</tr>
<tr>
<td>1</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
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<td>3</td>
<td>50%</td>
<td>25</td>
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<td>4</td>
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<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>8</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
<tr>
<td>10</td>
<td>50%</td>
<td>25</td>
<td>50%</td>
<td>4</td>
</tr>
</tbody>
</table>
CE tasks

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Mode</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>[11.92; 13.22]</td>
<td>[13; 14]</td>
<td>[14; 15]</td>
<td>[2.35; 1.98]^a</td>
<td>[6; 8]</td>
<td>[15; 18]</td>
<td>169</td>
</tr>
<tr>
<td>C3</td>
<td>[10.25; 11.80]</td>
<td>[10; 12]</td>
<td>[8; 10]</td>
<td>[3.16; 2.82]^a</td>
<td>&lt; 4</td>
<td>&gt; 25</td>
<td>169</td>
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</tbody>
</table>
### CE tasks

<table>
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<th>Mean</th>
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<th>Mode</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
<th>Obs</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>[11.92; 13.22]</td>
<td>[13; 14]</td>
<td>[14; 15]</td>
<td>[2.35; 1.98]$^a$</td>
<td>[6; 8]</td>
<td>[15; 18]</td>
<td>169</td>
</tr>
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<td>C3</td>
<td>[10.25; 11.80]</td>
<td>[10; 12]</td>
<td>[8; 10]</td>
<td>[3.16; 2.82]$^a$</td>
<td>&lt; 4</td>
<td>&gt; 25</td>
<td>169</td>
</tr>
</tbody>
</table>

Figure 1: Proportion of safe choices (CE tasks) and predictions
RQ2: What is the degree of ambiguity aversion?

(a) How do risk and model uncertainty aversion differ?

Result 2: (needs to be confirmed)
Subjects are both risk and model uncertainty averse. Model uncertainty aversion is stronger than risk aversion.
4. Structural estimations
Double MPL tasks

- Decision tables
- **Risky tasks** à la Holt and Laury (2002)
- **Model uncertain tasks**
- For each task:
  - 2 options (A and B)
  - 10 decisions made → switching point identified
RQ2: What is the degree of ambiguity aversion?

- Expo-Power function (Saha, 1993):
  \[ U(x) = \frac{1 - \exp\left(-ax^{1-r}\right)}{a} \]

- DARA and IRRA when \( a \geq 0 \) and \( r \geq 0 \)

- Special cases:
  - CRRA when \( a = 0 \): \( U(x) = x^{1-r} \)
  - CARA when \( r = 0 \): \( U(x) = \frac{1 - \exp(-ax)}{a} \)
RQ2: What is the degree of ambiguity aversion?

(b) What are the degrees of risk and model uncertainty aversion?

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>v</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CRRA</td>
<td>EP</td>
</tr>
<tr>
<td>a</td>
<td>0.0294***</td>
<td>(0.00215)</td>
</tr>
<tr>
<td>r</td>
<td>0.279***</td>
<td>(0.0119)</td>
</tr>
<tr>
<td>noise parameter</td>
<td>0.103***</td>
<td>(0.00327)</td>
</tr>
<tr>
<td>Observations</td>
<td>5320</td>
<td>5320</td>
</tr>
<tr>
<td>Loglikelihood</td>
<td>-1550.3</td>
<td>-1516.8</td>
</tr>
</tbody>
</table>

Notes: Luce error specification is used in the estimation. Standard errors in parentheses. The EP risk specification

* $p-value < 0.05$, ** $p-value < 0.01$, *** $p-value < 0.001$

- Model uncertainty aversion stronger than risk aversion
- EP specification seems better: DARA and IRRA
RQ2-3: Characterizing ambiguity attitude

Characterizing risk and model uncertainty attitudes

Figure 2: Absolute (left) and relative (right) risk and model uncertainty aversion using EP estimates (95% confidence in grey).
RQ2-3: Characterizing ambiguity attitude

- Easy to compute the degree of ambiguity aversion
- Remember $\phi \equiv v \circ u^{-1}$

![Graph of absolute ambiguity aversion on the left and relative ambiguity aversion on the right.](image)

Figure 3: Absolute (left) and relative (right) ambiguity aversion obtained with EP function estimates
RQ2-3: Characterizing ambiguity attitude

- Easy to compute the degree of ambiguity aversion
- Remember $\phi \equiv v \circ u^{-1}$

![Graphs showing absolute and relative ambiguity aversion](image)

Figure 3: Absolute (left) and relative (right) ambiguity aversion obtained with EP function estimates

Result 3:

Decreasing absolute ambiguity aversion (DAAA) and constant relative ambiguity aversion (CRAA)
4. Conclusion
Objective: characterize ambiguity aversion
4. Conclusion

Objective: characterize ambiguity aversion

- Individuals exhibit different attitudes to different types of uncertainty
4. Conclusion

Objective: characterize ambiguity aversion

- Individuals exhibit different attitudes to different types of uncertainty
- When the setup is simple enough:
  - subjects reduce compound risk (objective probabilities)
  - subjects do not reduce compound uncertainty (subjective probabilities)
Objective: characterize ambiguity aversion

- Individuals exhibit different attitudes to different types of uncertainty
- When the setup is simple enough:
  - subjects reduce compound risk (objective probabilities)
  - subjects do not reduce compound uncertainty (subjective probabilities)
- Subjects are more averse to model uncertainty than to risk
  \[\rightarrow\] strongly correlated with ambiguity aversion
Objective: characterize ambiguity aversion

- Individuals exhibit different attitudes to different types of uncertainty
- When the setup is simple enough:
  - subjects reduce compound risk (objective probabilities)
  - subjects do not reduce compound uncertainty (subjective probabilities)
- Subjects are more averse to model uncertainty than to risk
  → strongly correlated with ambiguity aversion
- Evidence of decreasing absolute ambiguity aversion (DAAA)
Thank you
email: loic.berger@feem.it

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RISICO ERC-2013-336703
Art from the lab...

email: loic.berger@feem.it
Art from the lab...

email: loic.berger@feem.it
The randomness device

- Ideally, absent of experts’ information: perfect ignorance → infinite number of balls? Infeasible...

- Randomness device: total number of balls in Urn 3 itself unknown, and comprised between 1 and 100 → Total number of potential objectives models: 3045 (= cardinality of Farey sequence of order 100)

| $N$ | Set of possible models: $M_N = \{P(r)\}$ | $|M_N|$ |
|-----|--------------------------------------|-------|
| 1   | $\{0\}$                              | $\frac{1}{1}$ | 2     |
| 2   | $\{0, \frac{1}{2}\}$                | $\frac{1}{1}$ | 3     |
| 3   | $\{0, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}\}$ | $\frac{1}{1}$ | 5     |
| 4   | $\{0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}\}$ | $\frac{1}{1}$ | 7     |
| 5   | $\{0, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{2}{7}, \frac{3}{7}, \frac{3}{4}, \frac{4}{5}\}$ | $\frac{1}{1}$ | 11    |
| 6   | $\{0, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{2}{7}, \frac{3}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}\}$ | $\frac{1}{1}$ | 13    |
| 7   | $\{0, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{1}{2}, \frac{4}{7}, \frac{3}{7}, \frac{3}{5}, \frac{2}{7}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}\}$ | $\frac{1}{1}$ | 19    |
| 8   | $\{0, \frac{1}{8}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{2}{5}, \frac{3}{5}, \frac{1}{2}, \frac{4}{7}, \frac{3}{7}, \frac{3}{5}, \frac{2}{7}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{7}{8}\}$ | $\frac{1}{1}$ | 23    |

$\Rightarrow M_{100} = \{P(r) \in \mathcal{F}_{100}\} \sim \{P(r) \in [0,1]\}$
RQ1: Robustness round COP21

The main experiment presented in the body of the paper is not immune to imperfections and biases. We conducted a robustness round of the experiment at the COP21, held in Paris in December 2015. Participants originated from and represented more than 100 countries and most were climate negotiators. In individual in-person interviews, we prompted respondents who volunteered for the study for their prior probability distribution over possible 2100 temperature increases as the result of current INDCs.

<table>
<thead>
<tr>
<th></th>
<th>$C1 = C2$</th>
<th></th>
<th>$C1 = C3$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>No</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected</td>
<td>30.5</td>
<td>32.5</td>
<td>44.3</td>
<td>18.7</td>
</tr>
<tr>
<td></td>
<td>(41.76%)</td>
<td>(27.47%)</td>
<td>(60.44%)</td>
<td>(8.79%)</td>
</tr>
<tr>
<td>Yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected</td>
<td>13.5</td>
<td>14.5</td>
<td>19.7</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>(6.59%)</td>
<td>(24.18%)</td>
<td>(9.89%)</td>
<td>(20.88%)</td>
</tr>
<tr>
<td>Total</td>
<td>44</td>
<td>47</td>
<td>64</td>
<td>27</td>
</tr>
<tr>
<td></td>
<td>(48.35%)</td>
<td>(51.65%)</td>
<td>(70.33%)</td>
<td>(29.67%)</td>
</tr>
</tbody>
</table>

Notes: Relative frequencies in parentheses. Chi-square test: 6.1e-4 Chi-square test: 1.1e-07
Robustness round

Logistic regressions:

- Dependent variable: Ambiguity neutrality ($C1 = C4$)
- Regressors:
  - reduction of compound risk ($C1 = C2$)
  - reduction of compound uncertainty ($C1 = C3$)

<table>
<thead>
<tr>
<th></th>
<th>Odds Ratio</th>
<th>Standard Error</th>
<th>Lower 95% Confidence Interval</th>
<th>Upper 95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C1 = C2$</td>
<td>5.573**</td>
<td>2.941</td>
<td>1.981</td>
<td>15.677</td>
</tr>
<tr>
<td></td>
<td>(1.793)</td>
<td>(1.174)</td>
<td>(0.497)</td>
<td>(6.472)</td>
</tr>
<tr>
<td>$C1 = C3$</td>
<td>14.514***</td>
<td>8.041</td>
<td>4.900</td>
<td>42.989</td>
</tr>
<tr>
<td></td>
<td>(10.671***)</td>
<td>(6.790)</td>
<td>(3.066)</td>
<td>(37.138)</td>
</tr>
</tbody>
</table>

Notes: Logistic regressions. Adjusted results in parentheses. Dependent variable: Ambiguity neutrality ($C1 = C4$). 91 observations.
* $p-value < 0.05$, ** $p-value < 0.01$, *** $p-value < 0.001$

- Probability of $C1 = C4$ is 31%
- it increases to 70% if $C1 = C3$, and decreases to 14% if $C1 \neq C3$
- it increases to 47% if $C1 = C2$, and decreases to 14% if $C1 \neq C2
Table B.1: Association between attitudes towards ambiguity, compound risk and model uncertainty

<table>
<thead>
<tr>
<th>Compound risk</th>
<th>Model uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 &gt; C2</td>
<td>C1 ≥ C2</td>
</tr>
<tr>
<td>C1 = C2</td>
<td>C1 ≤ C2</td>
</tr>
<tr>
<td>C1 &lt; C2</td>
<td>C1 &lt; C2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Total</th>
<th>Count</th>
<th>Expected</th>
<th>(Relative frequency)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 &gt; C4</td>
<td>33</td>
<td>23</td>
<td>(17.46%)</td>
</tr>
<tr>
<td>C1 = C4</td>
<td>134</td>
<td>93</td>
<td>(70.9%)</td>
</tr>
<tr>
<td>C1 &lt; C4</td>
<td>22</td>
<td>17</td>
<td>(11.64%)</td>
</tr>
</tbody>
</table>

Notes: First row represents results of the main experiment (N=189), second row represents results of robustness round (N=91). Bars represent 95% confidence levels.
RQ2-3: Characterizing ambiguity attitude

- **Direct estimation of $\phi$:**

<table>
<thead>
<tr>
<th></th>
<th>CRAA</th>
<th>EP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_\phi$</td>
<td>-1.802</td>
<td>-1.802</td>
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<tr>
<td></td>
<td>(0.9655)</td>
<td>(0.9655)</td>
</tr>
<tr>
<td>$r_\phi$</td>
<td>0.534***</td>
<td>0.86***</td>
</tr>
<tr>
<td></td>
<td>(0.0261)</td>
<td>(0.0452)</td>
</tr>
<tr>
<td>noise parameter</td>
<td>0.0476***</td>
<td>0.0363***</td>
</tr>
<tr>
<td></td>
<td>(0.00213)</td>
<td>(0.00184)</td>
</tr>
<tr>
<td>Observations</td>
<td>7570</td>
<td>7570</td>
</tr>
<tr>
<td>Loglikelihood</td>
<td>-3680.6</td>
<td>-3675.1</td>
</tr>
</tbody>
</table>

Standard errors in parentheses

* $p-value < 0.05$, ** $p-value < 0.01$, *** $p-value < 0.001$