Where does Popularity of the CES Originates From?

The Story of an Abandoned Research Question

J.I. Witajewski-Baltvilks

FEEM seminar, July 2014

America, beginning of 20th c.

American economists' recipe for healthy economy:

Fight Monopolists

1950

The Story of One Research Question

Wait!

The Story of One Research Question

PRODUCT HETEROGENEITY AND PUBLIC POLICY

By E. H. CHAMBERLIN
Harvard University

- Different people have different preferences =>
- Society needs variety of products =>
- Firms offer variety of products only if we allow for some degree of monopoly power

1975

Monopolistic Competition and Optimum Product Diversity

By Avinash K. Dixit and Joseph E. Stiglitz*

- Consider a representative consumer with preferences described by the CES utility function: $U = \left(\int x_i^{\rho}\, di\right)^{\frac{1}{\rho}}$
- Then consumer's demand for a good is positive even if its price is high
- Producer can charge price above production costs
- this allows firms to cover fixed cost of designing new good

The reaction to Dixit and Stiglitz paper:

Microeconomists:	Macroeconomists:
The model fails to explain the link between taste heterogeneity and love for variety (Pettengill (1979), Perloff and Salop (1985))	

The Story of One Research Question

The reaction to Dixit and Stiglitz paper:

Microeconomists:	Macroeconomists:
The model fails to explain the link between taste heterogeneity and love for variety (Pettengill (1979), Perloff and Salop (1985))	This is the new foundation for macroeconomic models!

Taste Heterogeneity

Let utility of agent i be

$$U_i = \left(\sum_{j=1}^{N} (\theta_{ij} x_{ij})^{\rho} \right)^{\frac{1}{\rho}}$$

- x_{ij} is the quantity of product j consumed by individual i
- θ_{ij} is the idiosyncratic taste parameter
- Taste heterogeneity: each consumer might have different valuation of product j

The agent chooses optimal consumption basket, $\mathbf{x}$ given his income, y and set of prices $\mathbf{p}$

The demand curve for good j:

$Q_j(\mathbf{p};y) = \int x_i(\mathbf{p};y)\,di$

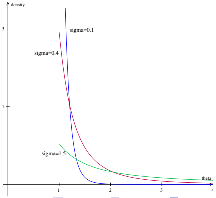
1. If $\rho < 1$

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -\frac{1}{1-\rho}\left(1-\rho\frac{E\left(\phi_j^2\right)}{E\left(\phi_j\right)}\right)$$

where$\phi_{ij} = \frac{p_j x_{ij}}{y}$, i.e. share of expenditure devoted for good j

2. If $\rho = 1$ (perfect substitutes) and $\ln(\theta_j) \sim \mathit{exponential}(\sigma)$

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -\left(1+\frac{1}{\sigma}\right)$$



Corollary 1

A positive represenatitive consumer exists if there is a rational preference relation such that the aggregate demand function is precisely the Walrasian demand function generated by this preference relation.

- Consider an economy with two goods - perfect substitutes
- Let θ_{ij} be the valuation of good j by individual i. Assume $\ln(\theta_j) \sim exponential(\sigma)$
- Then aggregate demand for good j is:

$$\log(q_j) = -\left(1 + \frac{1}{\sigma}\right)\log(p_j) + \log\left(\frac{y}{2}\right)$$

- Now consider an agent with $U = (0.5q_1^r + 0.5q_2^r)^{\frac{1}{r}}$ and $r = \left(\frac{\sigma}{2} + 1\right)^{-1}$
- his Walrasian deman is:

$$\log(q_j) = -\left(1 + \frac{1}{\sigma}\right)\log(p_j) + \log\left(\frac{y}{2}\right)$$

Corollary 1 and WITCH

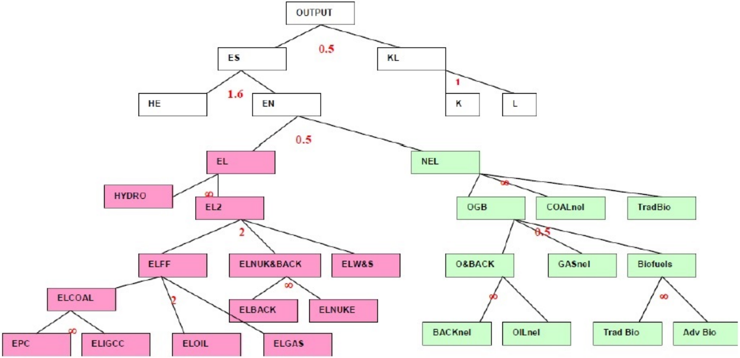


Figure 1: Production nest and the elasticity of substitution

Legenda: KL= Capital-labour aggregate; K = Capital invested in the production of final good; L = Labour; ES = Energy services; HE = Energy R&D capital; EN = Energy; EL = Electric energy; NEL = Non-electric energy; OGB = Oil, Backstop, Gas and Biofuel nest; ELFF = Fossil fuel electricity nest; W&S= Wind and Solar; ELj = Electricity generated with technology j (IGCC plus CCS, Oil, Coal, Gas, Backstop, Nuclear, Wind plus Solar); TradBiom= Traditional Biomass; TradBio= Traditional Biofuels; AdvBio= Advanced Biofuels

Young's endogenous growth model:

- N goods produced by N monopolists.
- A monopolists may reduce production costs by investing in R&D.
- A reduction of cost by a factor λ necessitates R&D spending of $F(\lambda) = fe^{\mu\lambda}$
- Monopolists optimization problem is:

$$\max_{p,\lambda} (p - c) Q - F(\lambda)$$

subject to $c = \frac{\varphi}{\lambda}$

Corollary 2 - uncertainty and R&D (2)

First order conditions imply:

$$\lambda = \frac{dQ_j}{dp_j}\frac{p_j}{Q_j}$$

Intuition:

- In equilibrium price has a constant mark-up over production costs: $p = m * c$
- R&D => costs reduction => lower price => higher market share => higher profit
- Incentives to perform R&D depends crucially on the respons of market share to changes in prices.

Digression Recall that

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -\frac{1}{1-\rho}\left(1 - \rho\frac{E\left(\phi_j^2\right)}{E\left(\phi_j\right)}\right)$$

whereϕ_{ij} is share of expenditure devoted for good j by individual i

Corollary 2 - uncertainty and R&D (3)

Young's endogenous growth + uncertainty:

- The R&D output is uncertain
- R&D spending of $F(\lambda) = fe^{\mu\lambda}$ brings cost reduction by $\theta\lambda$ where θ is random variable with $E(\theta) = 1$
- Monopolists optimization problem is:

$$\max_{p,\lambda} E\left[(p-c)\,Q\right] - F(\lambda)$$

subject to $c = \frac{\varphi}{\theta\lambda}$

The prediction:

$$\lambda = \frac{\rho}{1-\rho}\left(1 - \frac{E\left(\psi_j^2\right)}{E\left(\psi_j\right)}\right)$$

where $\psi_{ij} = \frac{\theta_{ij}^{\frac{\rho}{1-\rho}}}{\sum_{k=1}^{N} \theta_{ik}^{\frac{\rho}{1-\rho}}}$

- For $Var\left(\psi_{ij}\right) = 0$ (no uncertainty), $\lambda = \frac{\rho}{1-\rho}\left(1 - \frac{1}{N}\right)$
- As uncertainty grows, firms invest less in R&D and (average) technological change is slower
- The result holds despite risk neutrality of firms.

Corollary 3 - growth and inequality

Young's endogenous growth + income inequality:

- Consider a dynamic model
- In addition allow income inequality: different consumers have different income.

Then:

$$\frac{\lambda_{jt}}{\lambda_{jt-1}} = \frac{\rho}{1-\rho}\left(1 - \frac{E\left(\phi_j^2 y\right)}{E\left(\phi_j y\right)}\right)$$

- No symmetric equilibrium and no analytical solution
- simulation result: quality of goods that are favoured by rich consumers grows faster than quality of goods that are favoured by the poor.

Conclusions

1. For models with symmetric equilibria: elasticity of demand can be expresses as a simple function of taste heterogeneity
2. For models with symmetric equilibria and CES utility: elasticity of substitution can be expresses as a simple function of taste heterogeneity.
3. If we assume that representative consumer see all goods as perfect substitutes, then we implicitely assume all consumers have identical valuation of each good.
4. As heterogeneity increases, technological progress slows down.
5. As uncertainty increases, R&D investment falls even if firms are risk neutral
6. Quality of goods favoured by rich consumers grows faster than quality of goods favoured by the poor.

Derivations

1. If $\rho < 1$

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -\frac{1}{1-\rho}\left(1-\rho\frac{E\left(\phi_j^2\right)}{E\left(\phi_j\right)}\right)$$

where$\phi_{ij} = \frac{p_j x_{ij}}{y}$, i.e. share of expenditure devoted for good j

2. If $\rho = 1$ (perfect substitutes) and $\ln\left(\theta_j\right) \sim exponential(\sigma)$

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -\left(1+\frac{1}{\sigma}\right)$$

Derivations

The demand curve for good j:
$Q_j(\mathbf{p}; y) = \int x_i(\mathbf{p}; y)\, di$

1. If $\rho < 1$

$$x_{ij} = \frac{\left(\theta_{ij}/p_j\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\theta_{ik}/p_k\right)^{\frac{\rho}{1-\rho}}} y p_j^{-1}$$

$$Q_j = \int \int \ldots \int \frac{\left(\theta_{ij}/p_j\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\theta_{ik}/p_k\right)^{\frac{\rho}{1-\rho}}} y p_j^{-1} g\left(\underline{\theta}\right) d\underline{\theta}$$

Derivations

The demand curve for good j:

$$Q_j = E\left[\frac{\left(\theta_{ij}/p_j\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\theta_{ik}/p_k\right)^{\frac{\rho}{1-\rho}}}\right] y p_j^{-1}$$

Let $\omega = \frac{\left(\theta_{ij}/p_j\right)^{\frac{\rho}{1-\rho}}}{\sum_k \left(\theta_{ik}/p_k\right)^{\frac{\rho}{1-\rho}}}$

The demand curve for good j:

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = \frac{-E\left[\omega\right] y p_j^{-1} + E\left[\frac{d\omega}{dp_j}\right] y}{E\left[\omega\right] y p_j^{-1}}$$

$$\frac{dQ_j}{dp_j}\frac{p_j}{Q_j} = -1 + \frac{E\left[\frac{d\omega}{dp_j}\frac{p_j}{\omega_j}\omega\right]}{E\left[\omega\right]}$$