FEEM Seminar Thursday May 29, 2014
Isola San Giorgio Maggiore, Venice

« TU and NTU Implementations of Coalitional Stability in Integrated Assessment Models: Taking Stock »

Henry Tulkens
CORE, Université catholique de Louvain, Belgium
Visiting Professor, Università Ca' Foscari Venezia and Research Associate, FEEM

Partly joint work with Ulrike Kornek and Kai Lessman,
PIK, Potsdam, Germany

PART I
The TU vs NTU issue
in a recent (first) coalition models comparison

- Starting point: A long awaited initiative:

a comparison between the models, increasing in number, that deal with coalitions in climatic issues with game theoretical arguments,

and especially « Integrated Assessment Models » (IAM) » simulation models doing that.

- Potsdam Feb 2012 and Venice Jan 2013 workshops gathered authors of five IAMs dealing with coalitions, namely: CWS (Louvain, 2003), STACO (Wageningen, 2006), RICE (Binghampton, 2009), MICA (Potsdam, 2011) , and WITCH (Milan, 2013).

- The Venice workshop led to FEEM *Nota di Lavoro # 2014-5* by the (9 authored) paper Lessman et al.

- Methodological issues arose as to the treatment of utilities (TU vs NTU) in these models, dealt with in a companion paper by Kornek, Lessman and Tulkens (now available, dated 2014).

- Their results are not, thus far, included in the numerical models. So that one has currently the following ….

state of the art in coalition models comparison.

Who does what as of October 2013,

and reported on in FEEM *Nota di Lavoro* #5.2014 by Lessman et al.

	TU models		NTU models	
	No transfers	With transfers	No transfers	With transfers
Core stability	CWS RICE	CWS	RICE	Under way !
Potential* Internal stability	CWS STACO MICA	CWS STACO MICA	RICE	WITCH

* Only this stability concept is covered in Nota di Lavoro #5.2014

Given that the methodological points concerning TU vs NTU are fairly resolved,
but not yet having been put to numerical tests,
the rest of this seminar will preferably be devoted to reconsidering the comparison between the two parallel concepts of *Core Stability* and *Potential Internal Stability*

PART II:

A graphical representation of « potentially internal stability » (PIS) of *any* coalition with a comparison with *that coalition's* « core stability » (CS)

1. **Coalitional behaviors**

   a What a coalition can achieve, as a coalition

   b Externality and the possibility of defection

2. **Spotting a player's outside option on the diagram**

3. **Representing several players' outside options**

4. **Feasibility of IS for the coalition: the « ISI set »**

5. **The « PISI set »**

6. **Comparing with a « core stable » coalition**

Textbook reference for graphical representations of a 3 players game:
Mas Colell, Green and Whinston 1995, chapter 18, Appendix A pp. 673 & ff.

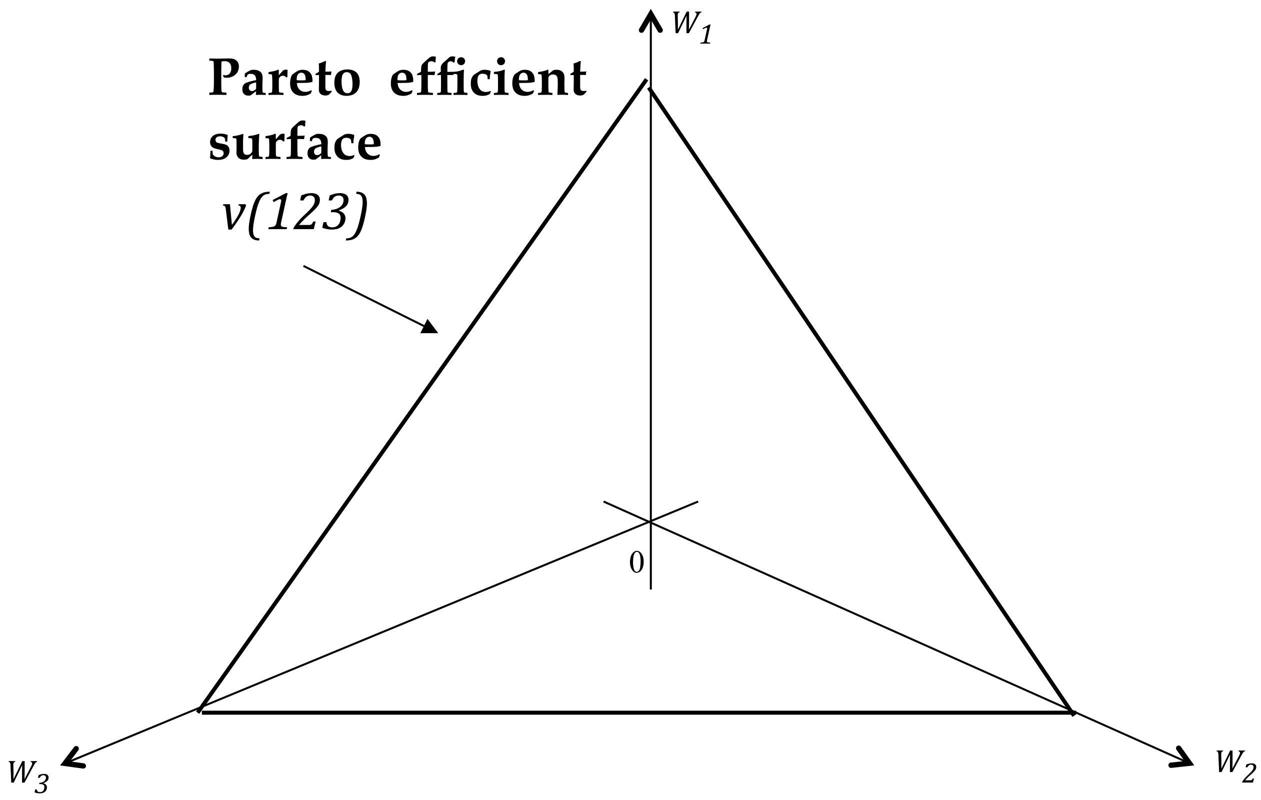
A technique applied and extended to the international environmental externality game in
Chander and Tulkens 2008, pp. 176-177 (« San Servolo » paper 2005)
and most explicitly in this paper, May 2014

***Model* : The standard international environmental externality game**

Not a symmetric game ! Full heterogeneity of players (i.e.countries).

Asumed TU, that is, quasi linear utilities, and convex damage functions.

1.1 Preliminaries: What a coalition can achieve, as a coalition

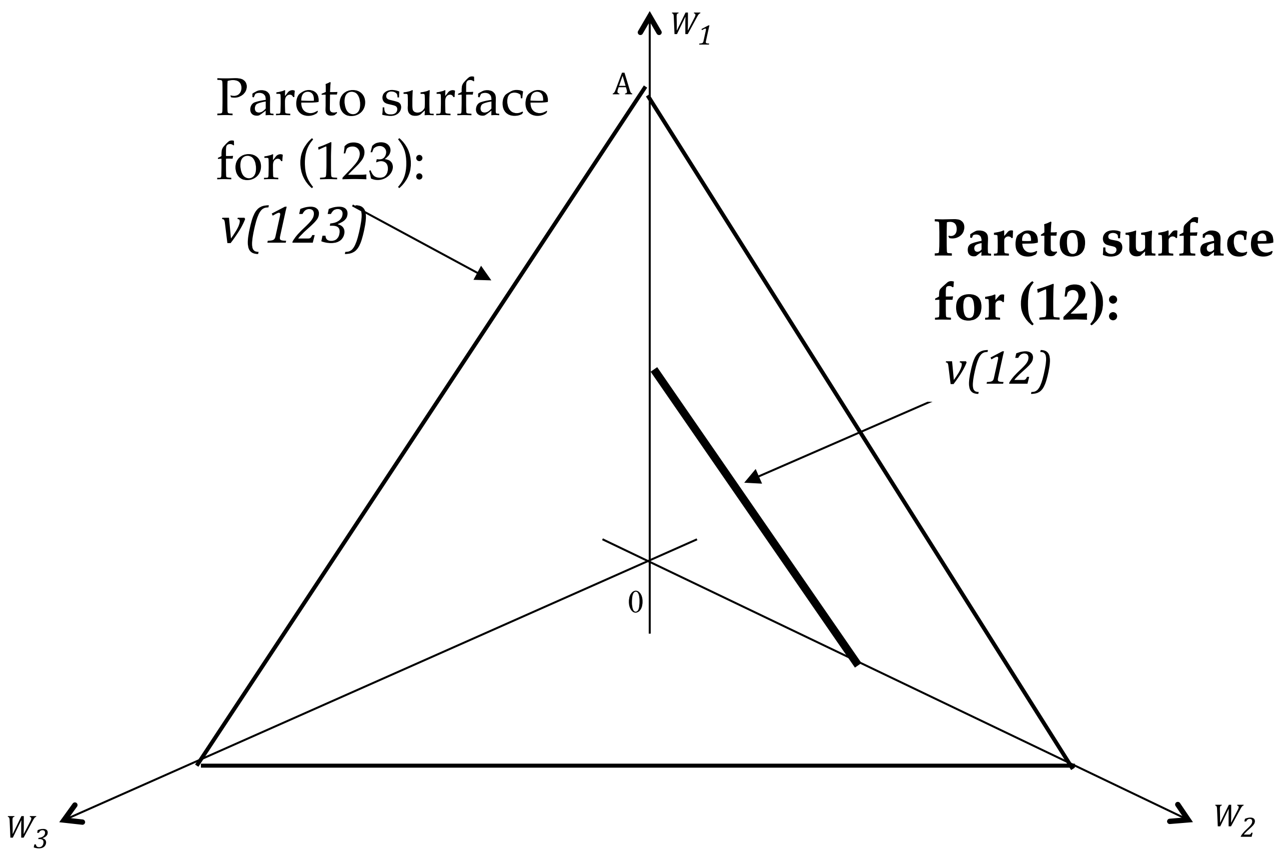


What coalition (123) can do

in a 3 players game (N,v)

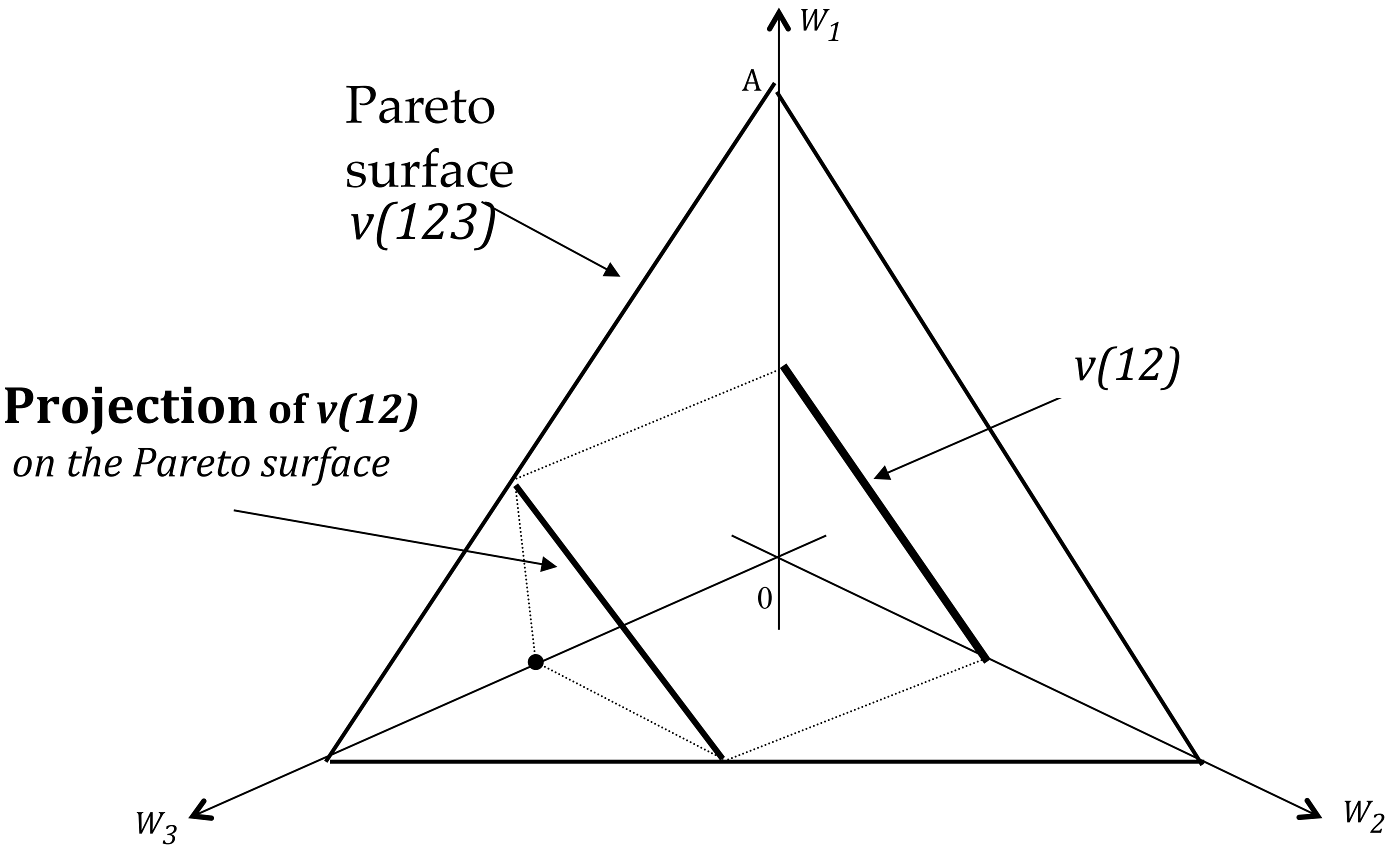
What coalition (12) could do

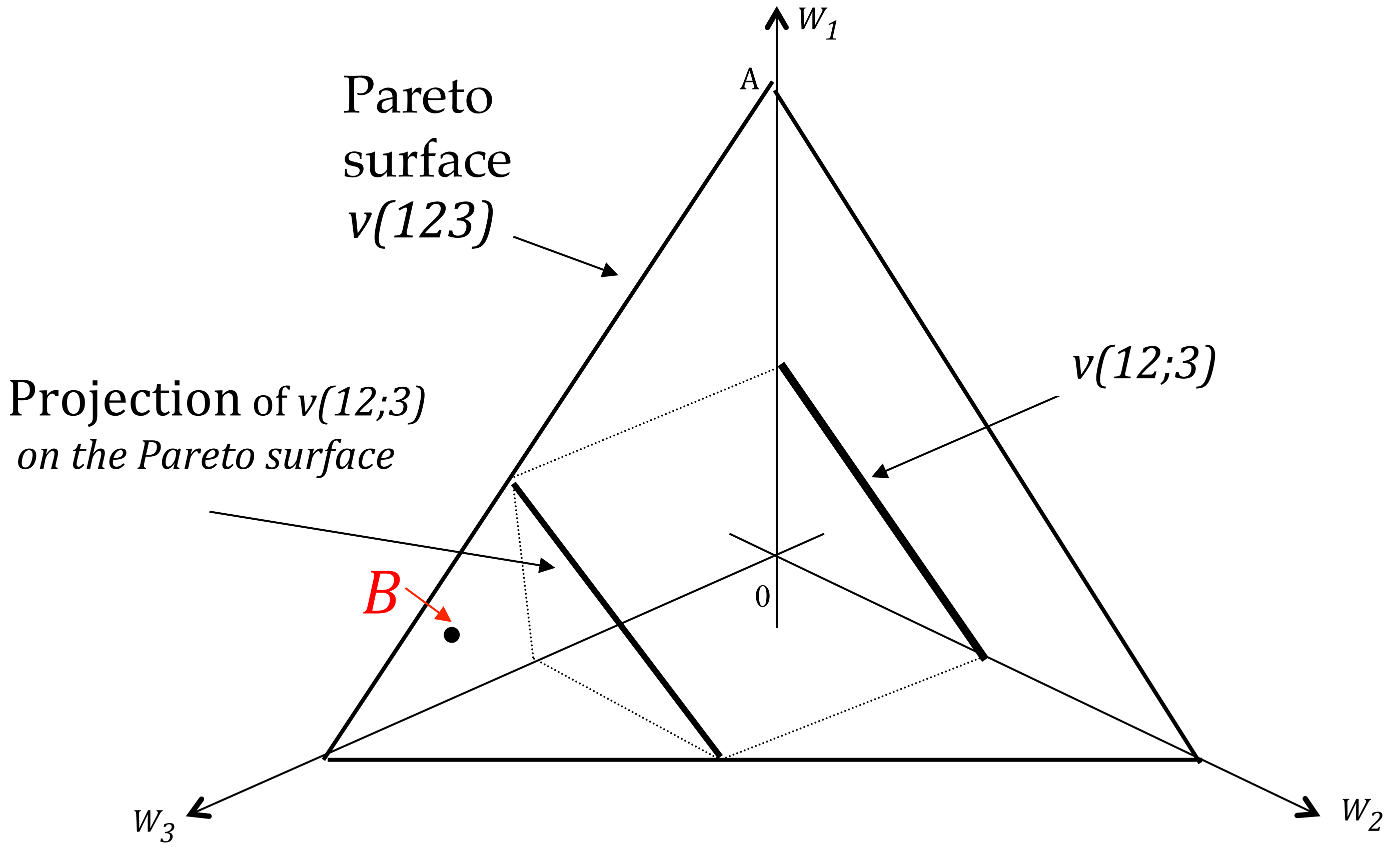
if it were a 2 players game (N,*v)* with *n* = 2.



A useful technique :

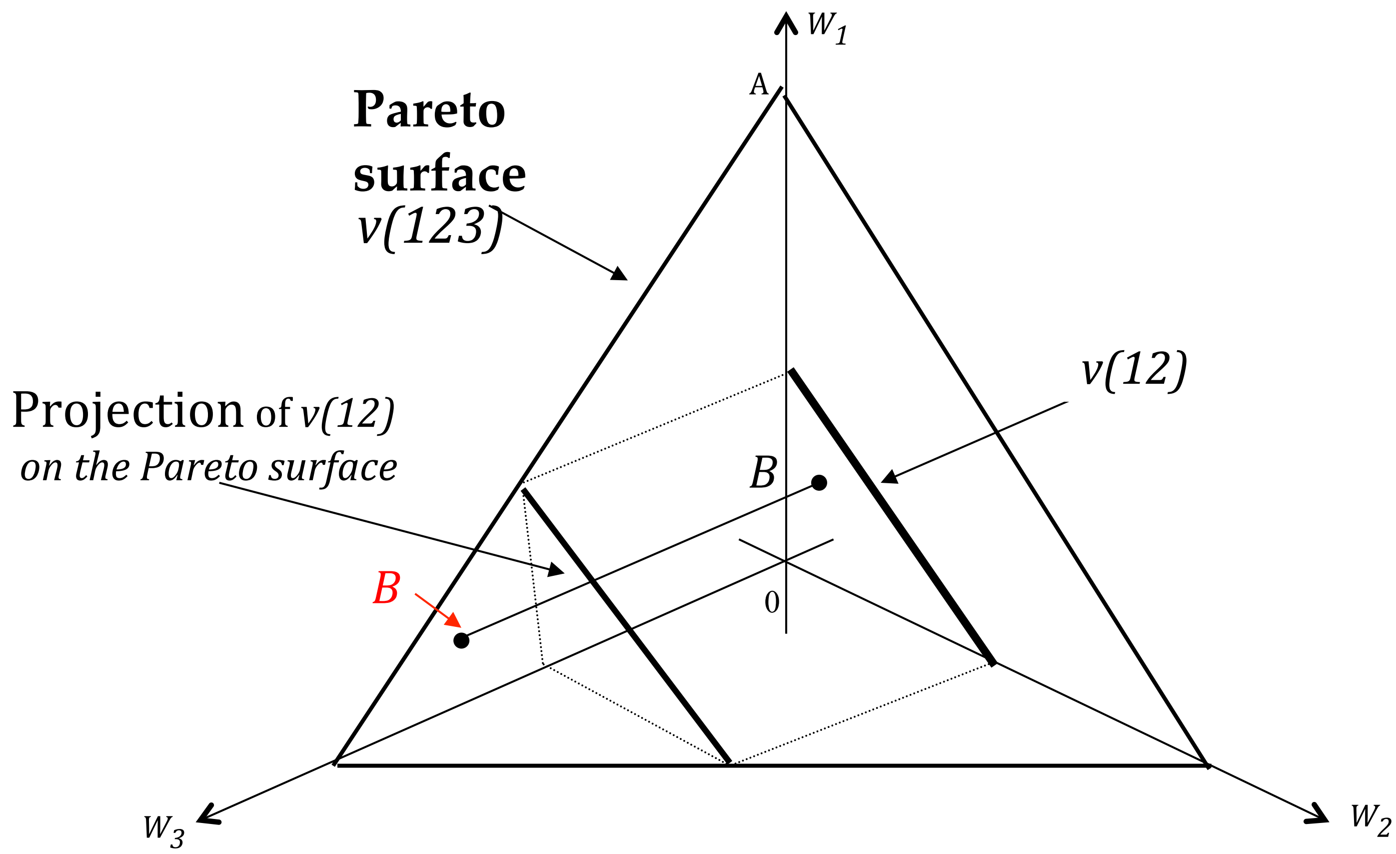
Projection of *v(12)* on the Pareto surface of *v(123)*





Important implication : « *B* », a Pareto efficient point for coalition (123),

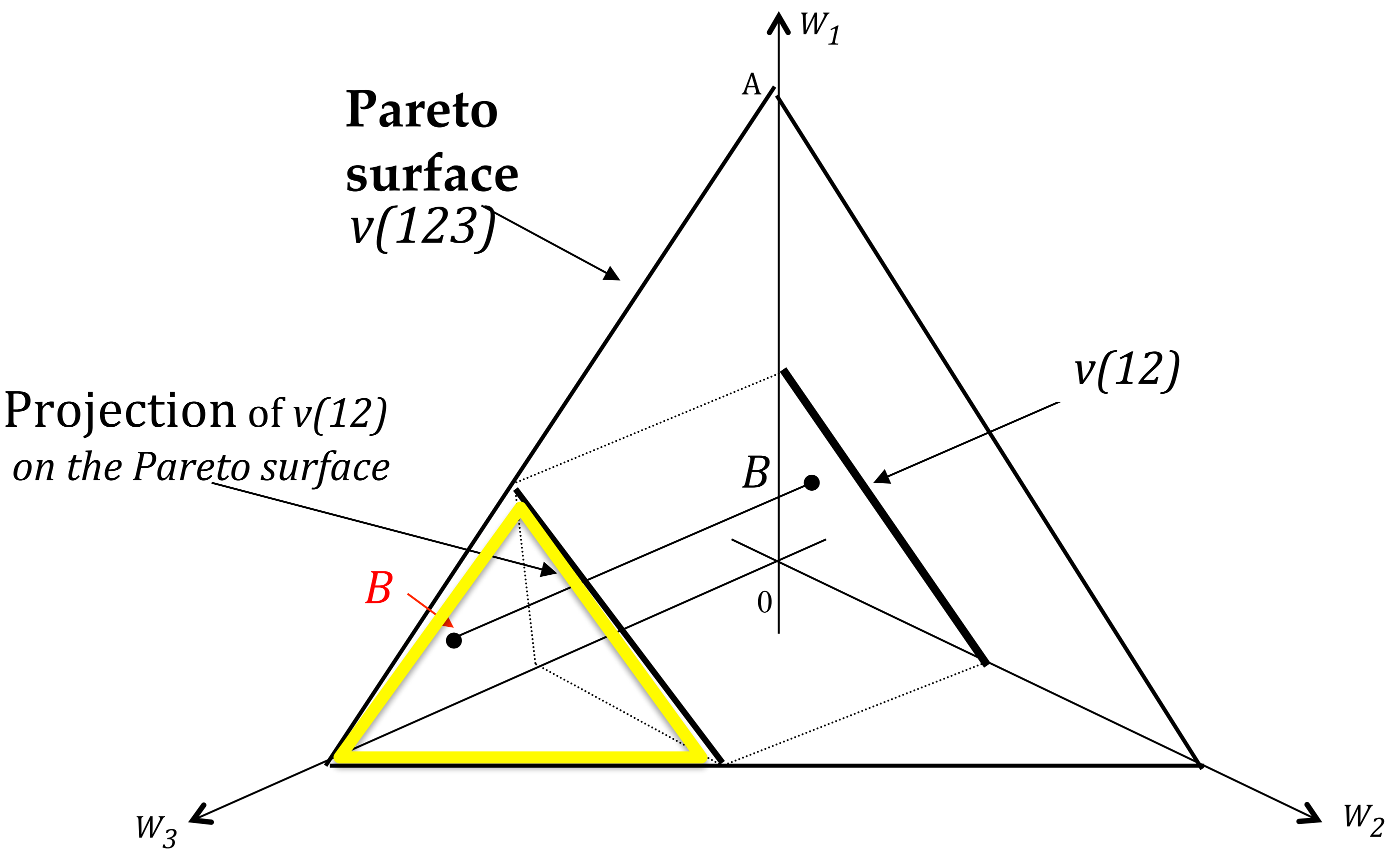
is rejected (« blocked ») by coalition (12) See WHY:



… because to point *B* there corresponds **point « B »**,

which is the ***projection back of*** *B* in the (W_1,W_2) space,

and « B » lies **below *v(12)***, that is, *below* what coalition (12) can do by itself.



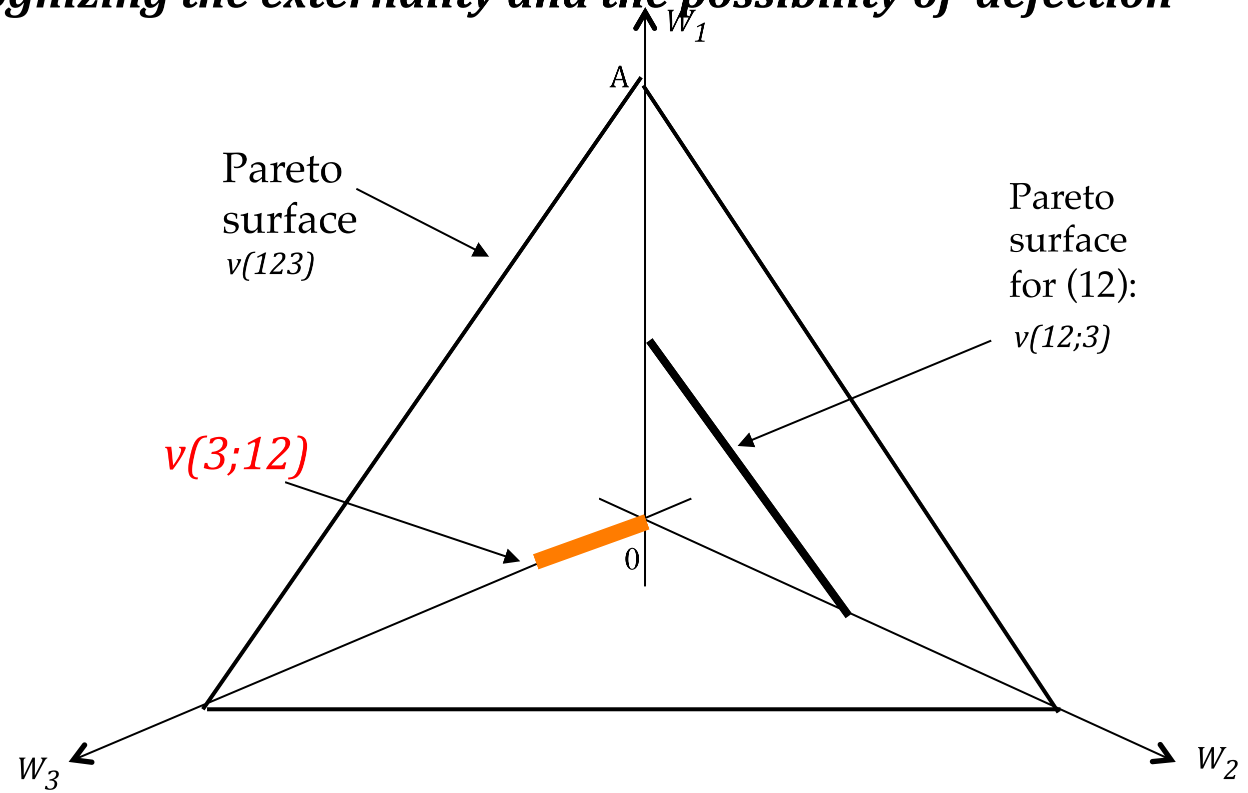
... because to point *B* there corresponds **point « B »**,
which is the ***projection back of*** *B* in the (W_1,W_2) space,

and « B » lies below *v(12)*, that is, *below* what coalition (12) can do by itself.

NOTE : This argument applies to all Pareto efficient points points in the **yellow triangle**

Less classical game theory:

1.2 Recognizing the externality and the possibility of defection



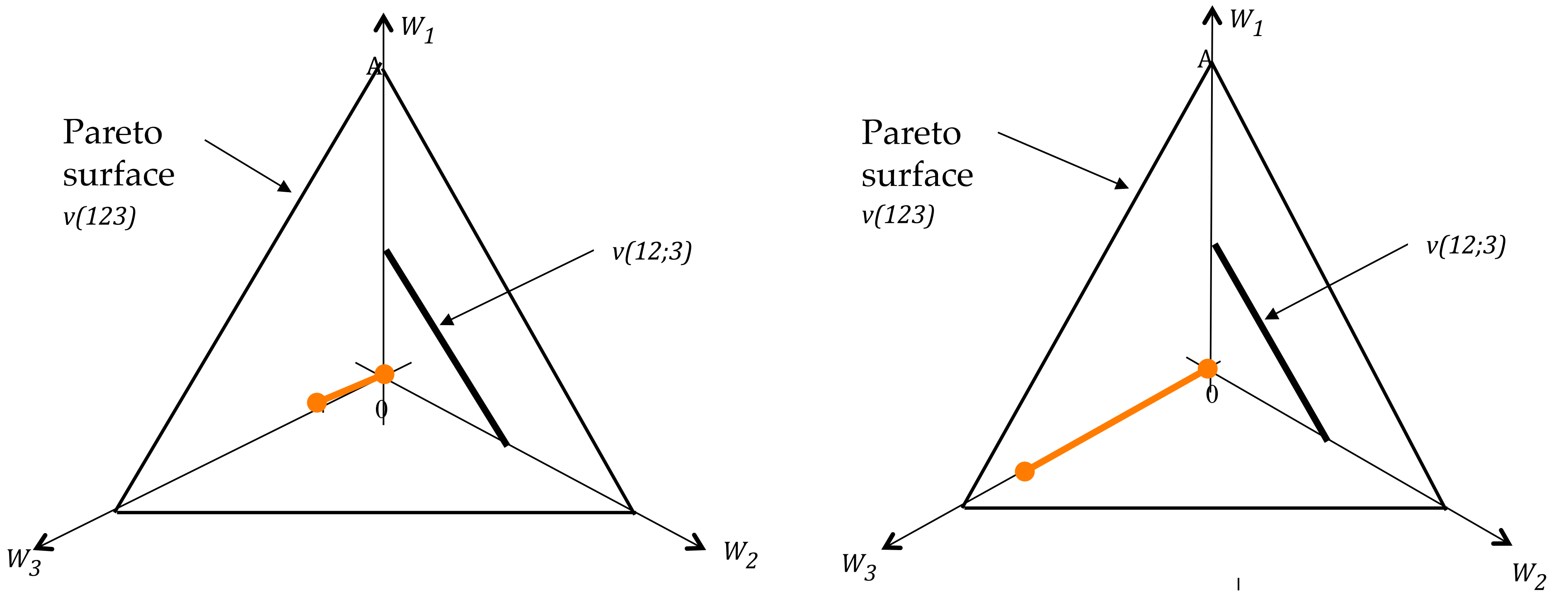
When defecting *from* coalition (123) *to* a PANE wrt coalition (12)

Partial **A**greement **N**ash **E**quilibrium **w**ith **r**espect **t**o coalition **(12)**

player 3's payoff is now to be written as *v(3;12)* : his payoff as a free rider.

Its amount is determined by his *sensitivity* to

the externality generated by coalition (12). ...INDEED...

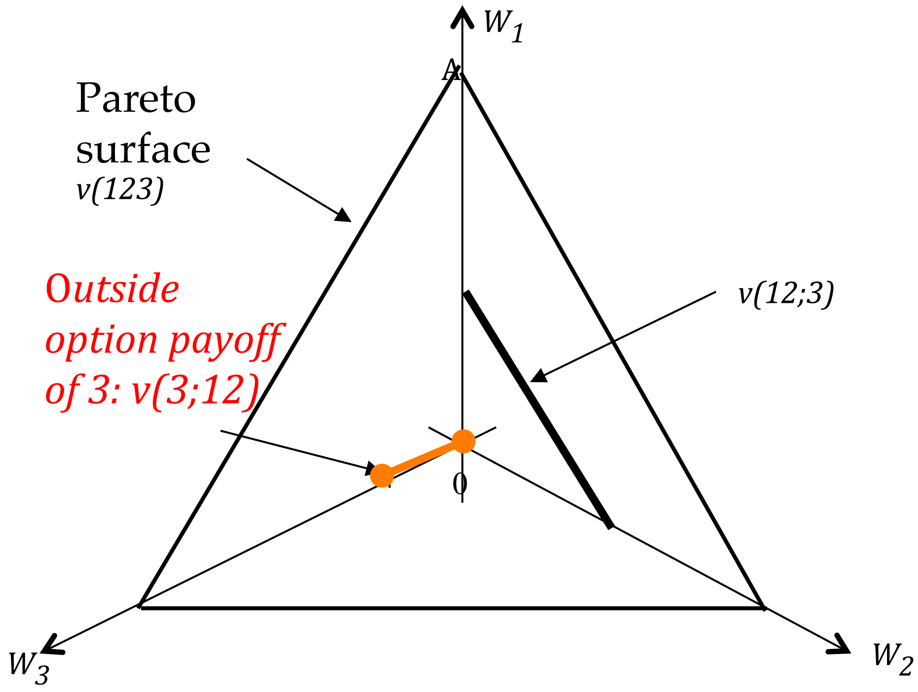


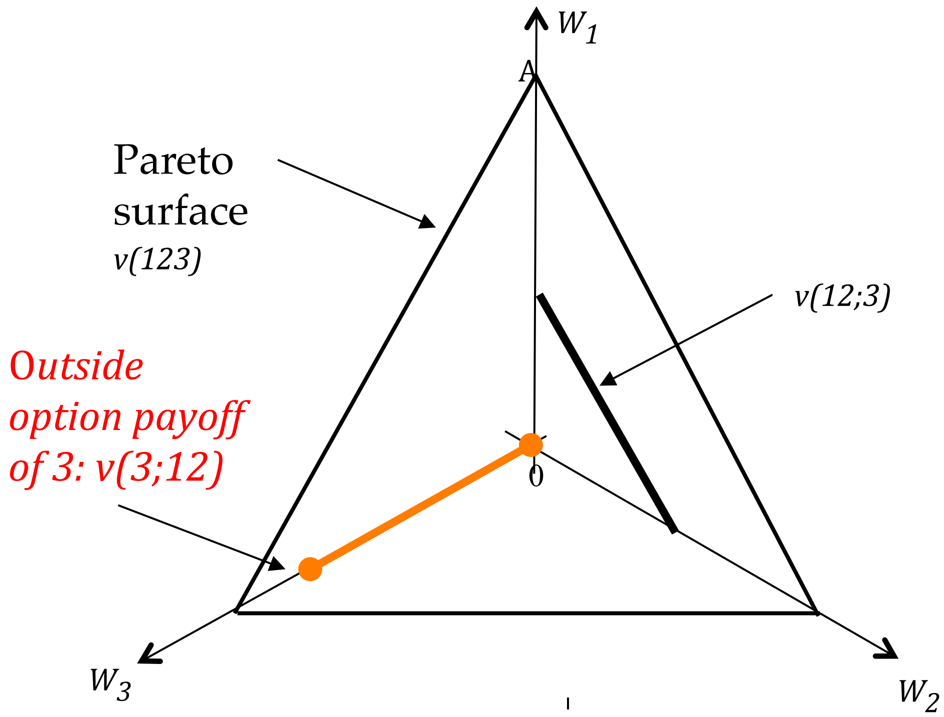
Two typical possible cases:

Player 3’s payoff varies little

Player 3’s payoff varies much.

2.a Spotting player 3's outside option payoff on the diagram





In either case, the point reached on the W_3 axis

measures Player's 3 « **outside option payoff** » at a PANE wrt (12)

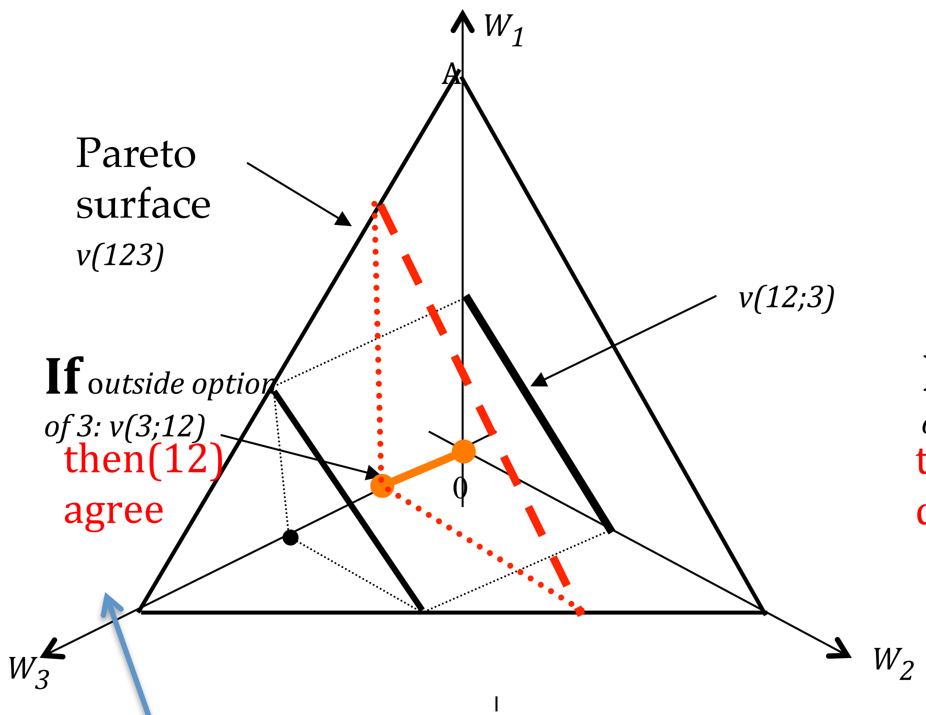
2.b Acceptability of player 3's outside option payoff within the (123) coalition

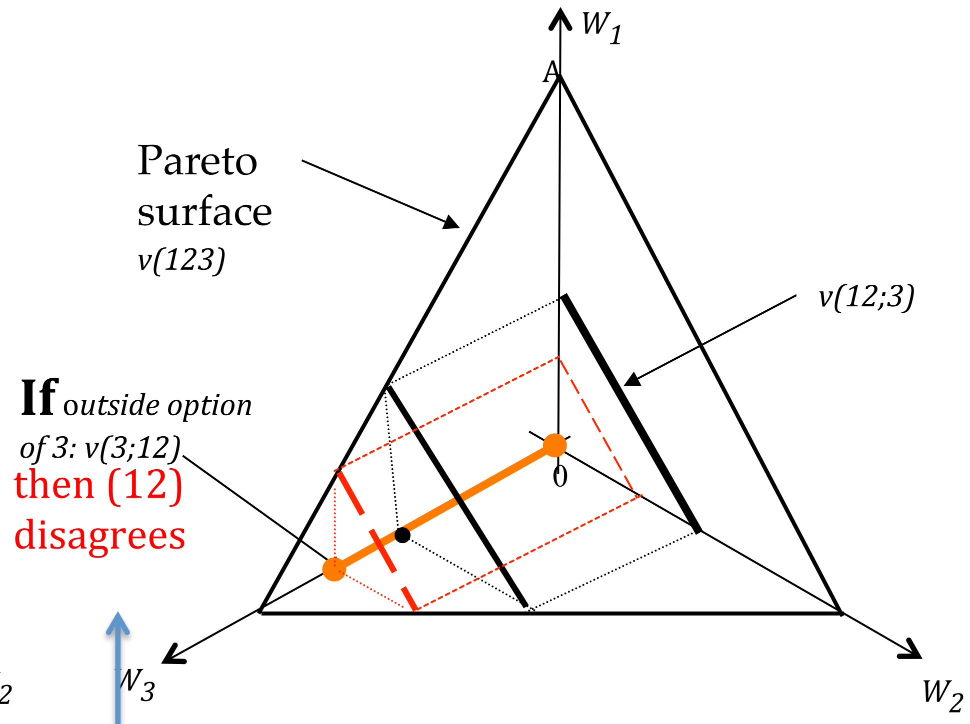
If, while remaining in the coalition (123),

player 3 requires as payoff his outside option at the Pareto surface

so as not to defect into a PANE wrt (12),

possible reactions by (12) are





possible reactions by (12) on request to grant 3 his outside option:

Agree

Disagree

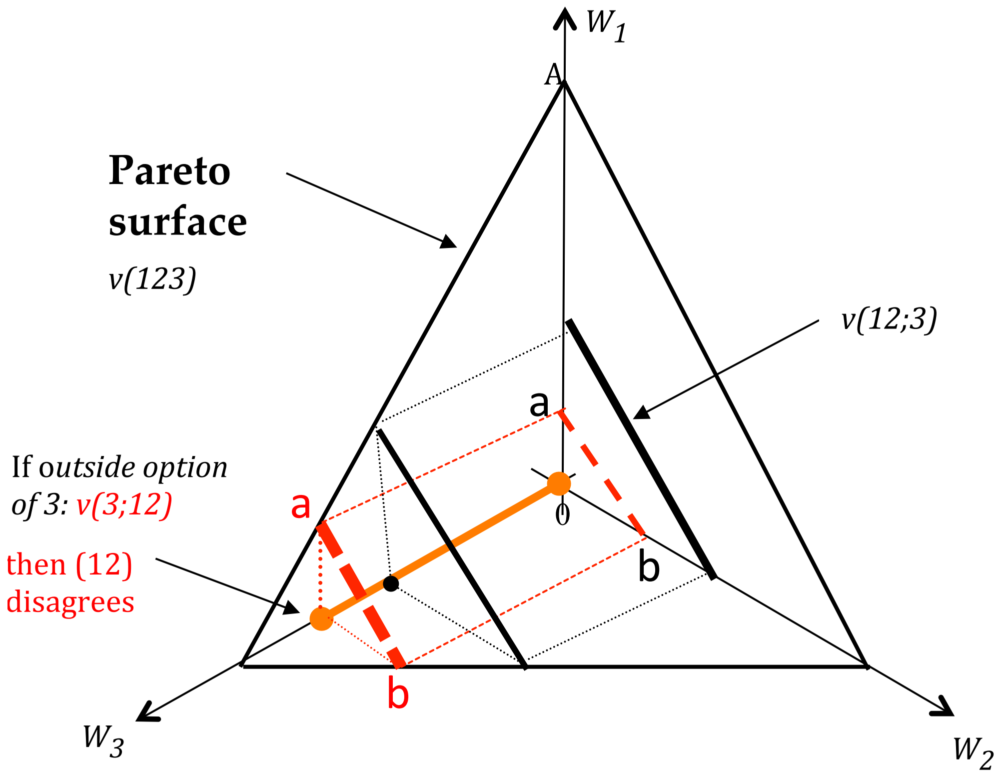
see WHY

Take the **case on the right:**

If 3 requires *v(3;12)* at Pareto,

***that is, any point on the line ab*,**

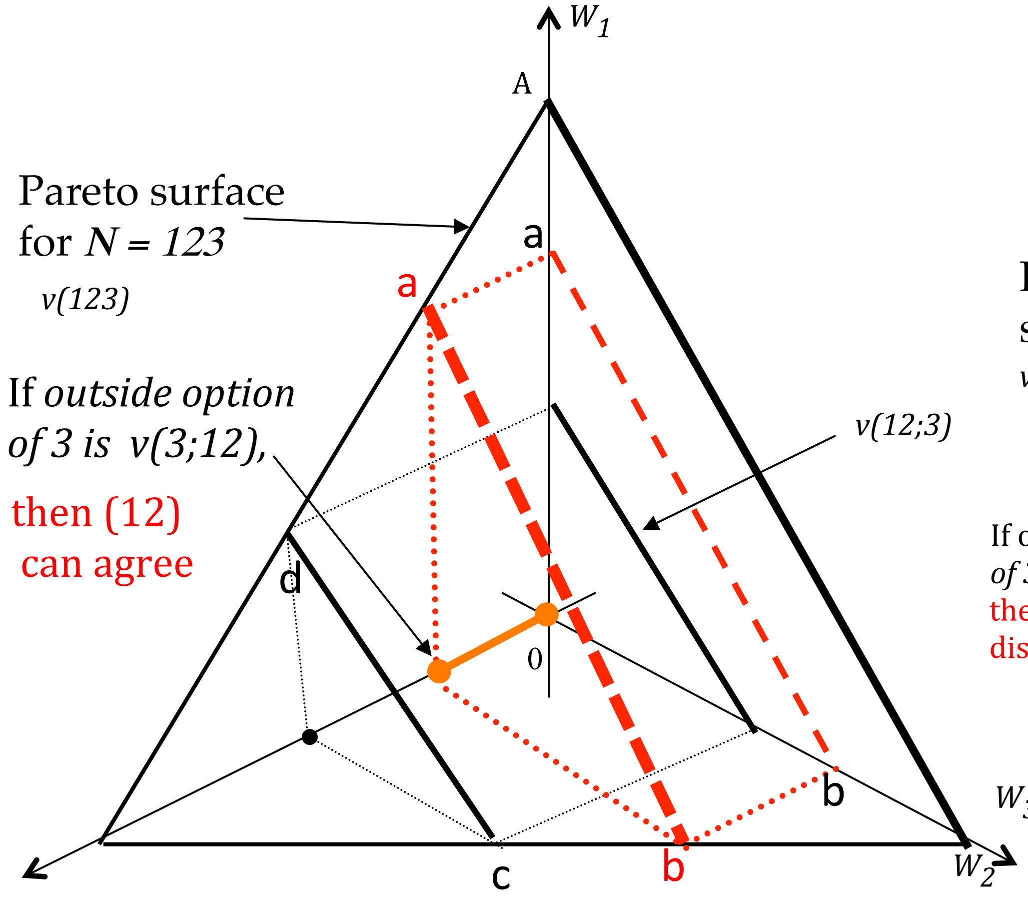
then it requires something visualized by line **ab**, which will be rejected by (12) because it is less than the amount *v(12;3)* that *(12) can achieve by itself.*

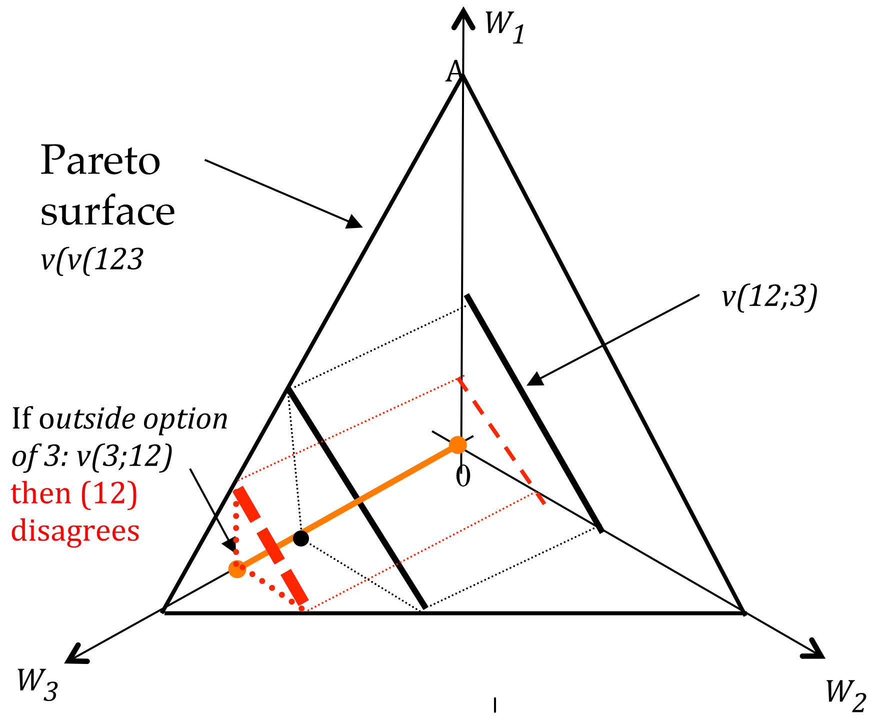


Thus, player 3 is demanding *a share* of the aggregate payoff *v(123)*

which is unacceptable for (12),
because (12) can do better for themselves by leaving 3 out and being on the line *v(12;3)*

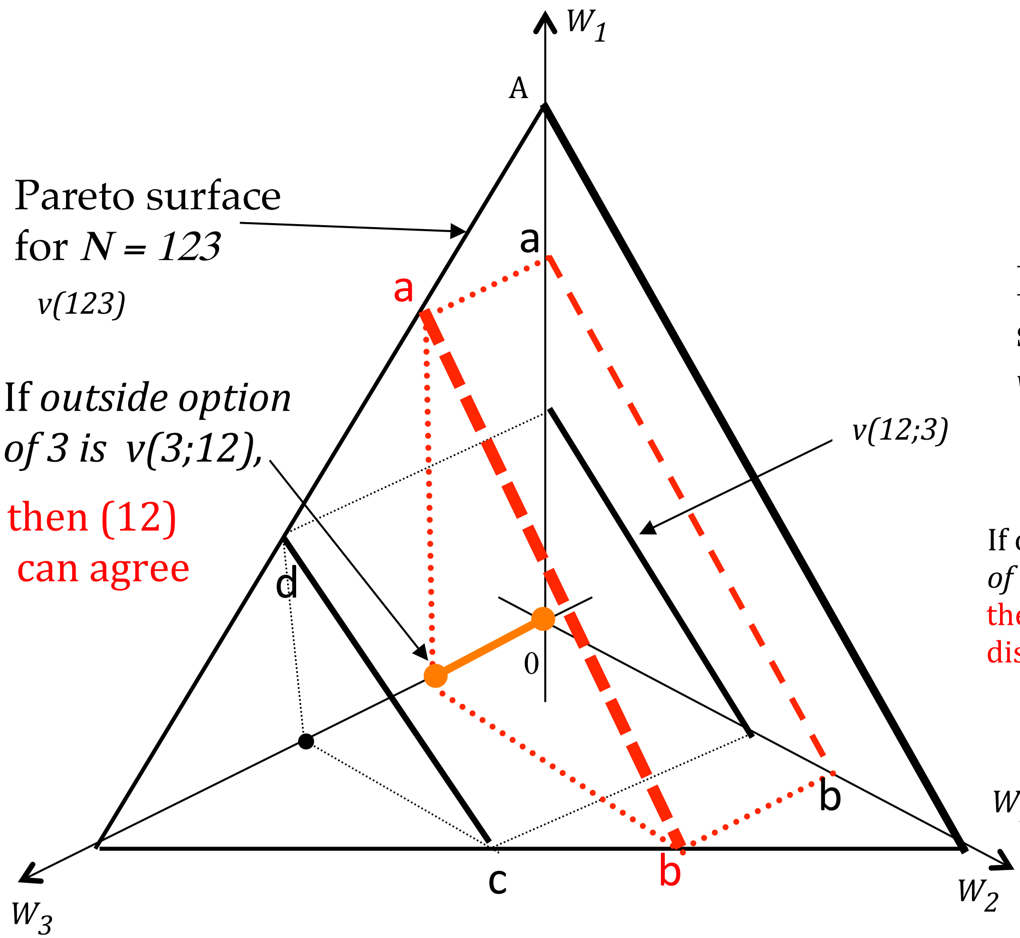
Thus **the coalition (123) is internally unstable**, and intead, **the PANE wrt (12) will prevail**.

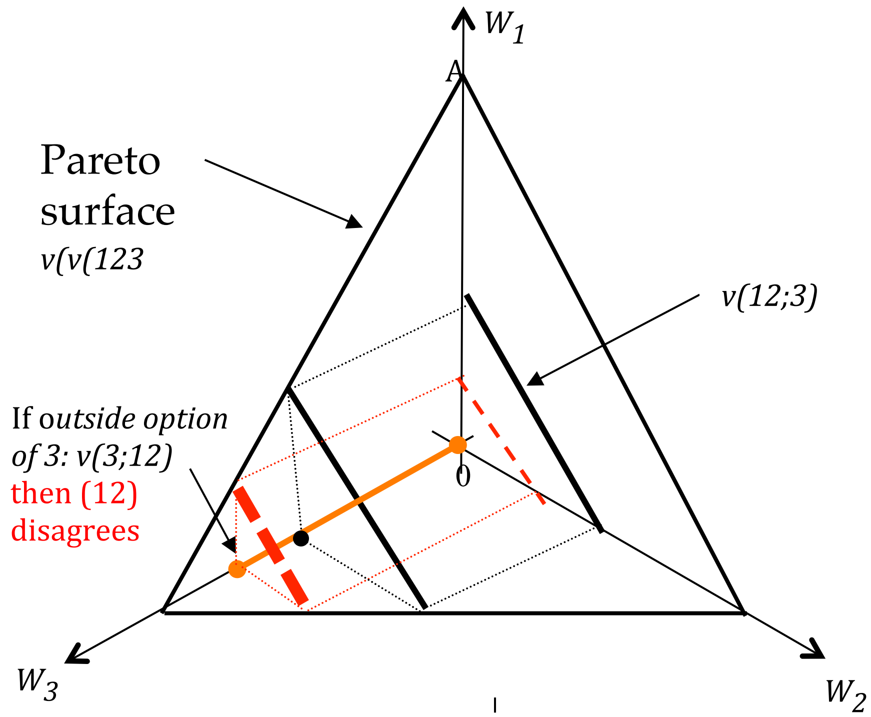




The **case on the left** , dealt with in the same way, leads to the ***opposite conclusion*** :

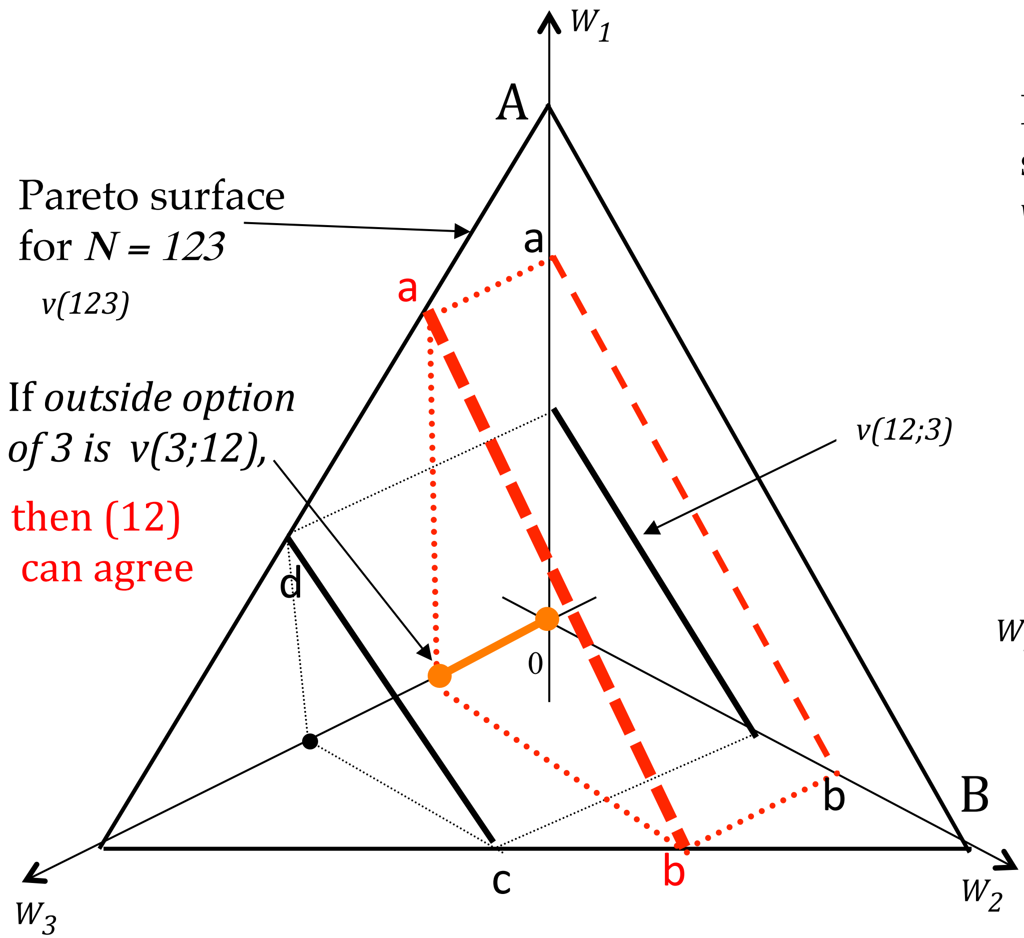
Player 3's outside option *v(3;12)* implies points on line ab on the Pareto surface whose projection **ab** lies ***above*** the line *v(12;3)* of what (12) can do by themselves. Then request can be met by (12) at Pareto, in terms of points within surface **abcd,** and **coalition (123) is internally stable**, *as far as player 3 is concerned.*

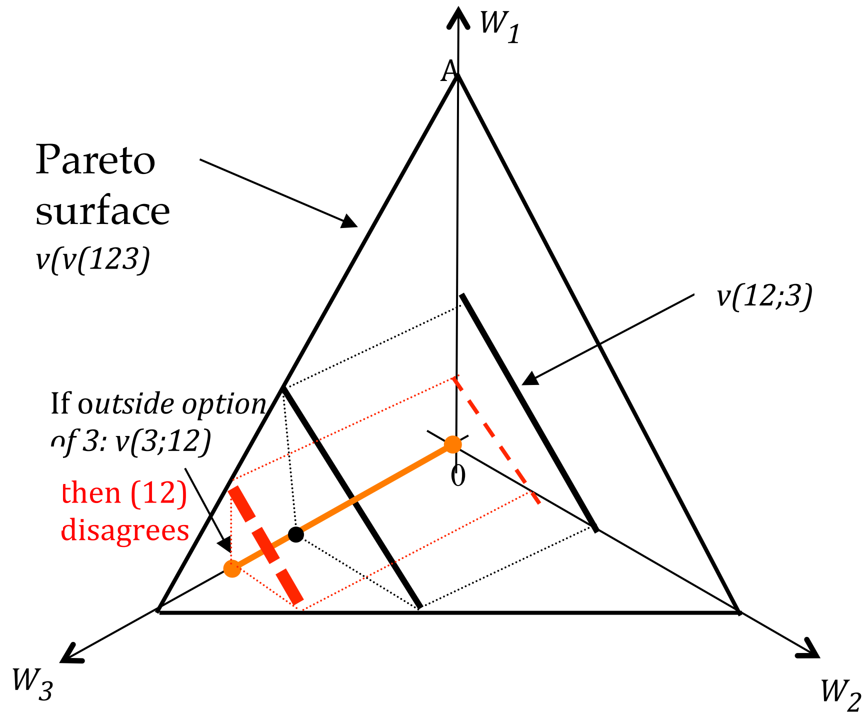




To summarize:

GRAPHICALLY, the free rider's requirement BITES on the Pareto surface





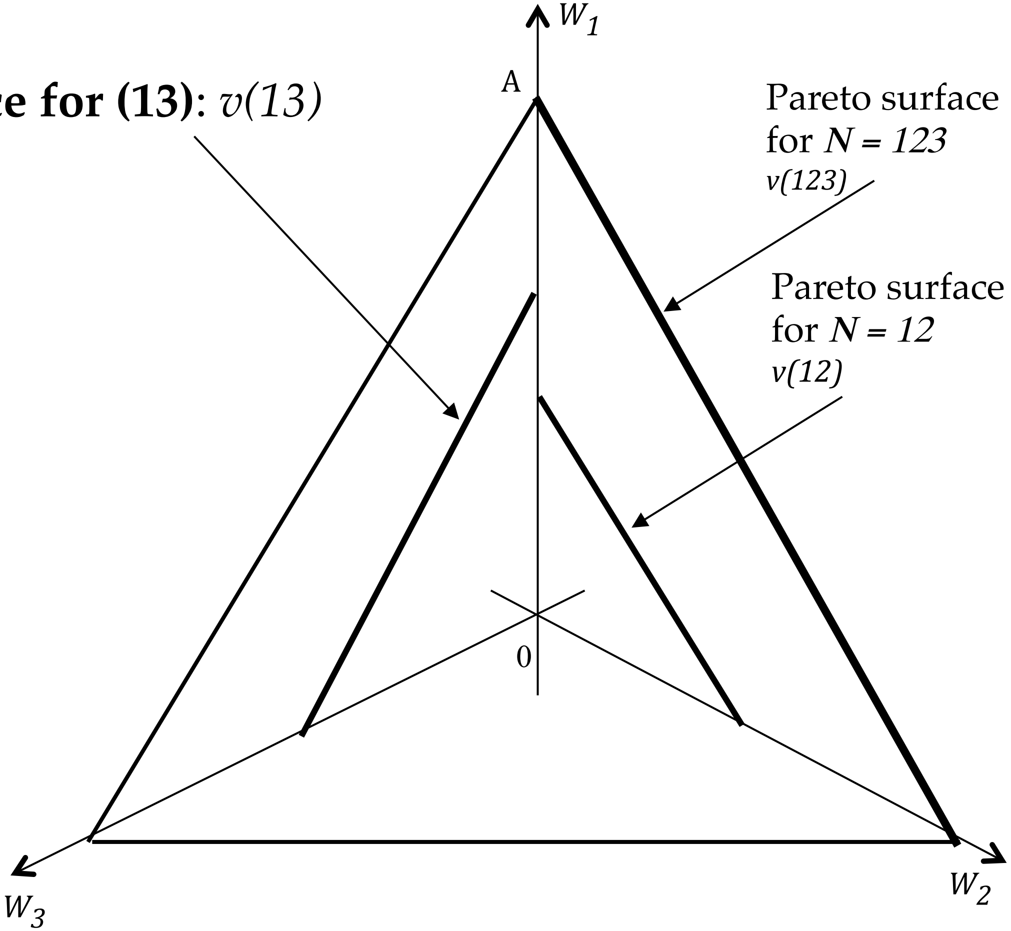
Details on ***BITES on the Pareto surface:***

the requirement excludes the area « aABb » for reason of outside option and leaves only the area « abcd » for sharing v(123)

3.1 Representing the outside option payoffs of a second player
- Preliminaries

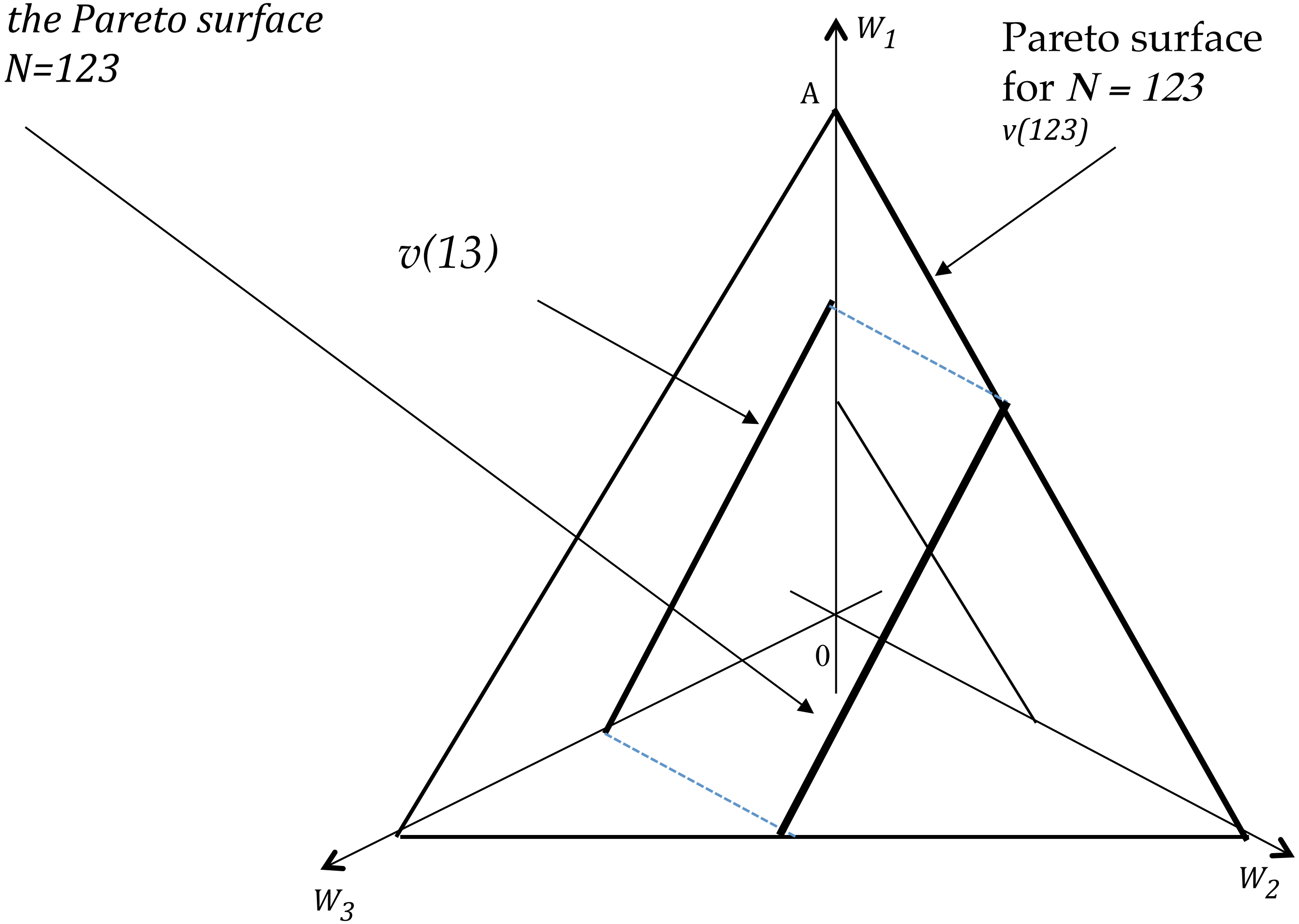
Starting all over again with the same game N= {1,2,3}
consider now what coalition (13) can do,
namely:

the **Pareto surface for (13)**: *v(13)*



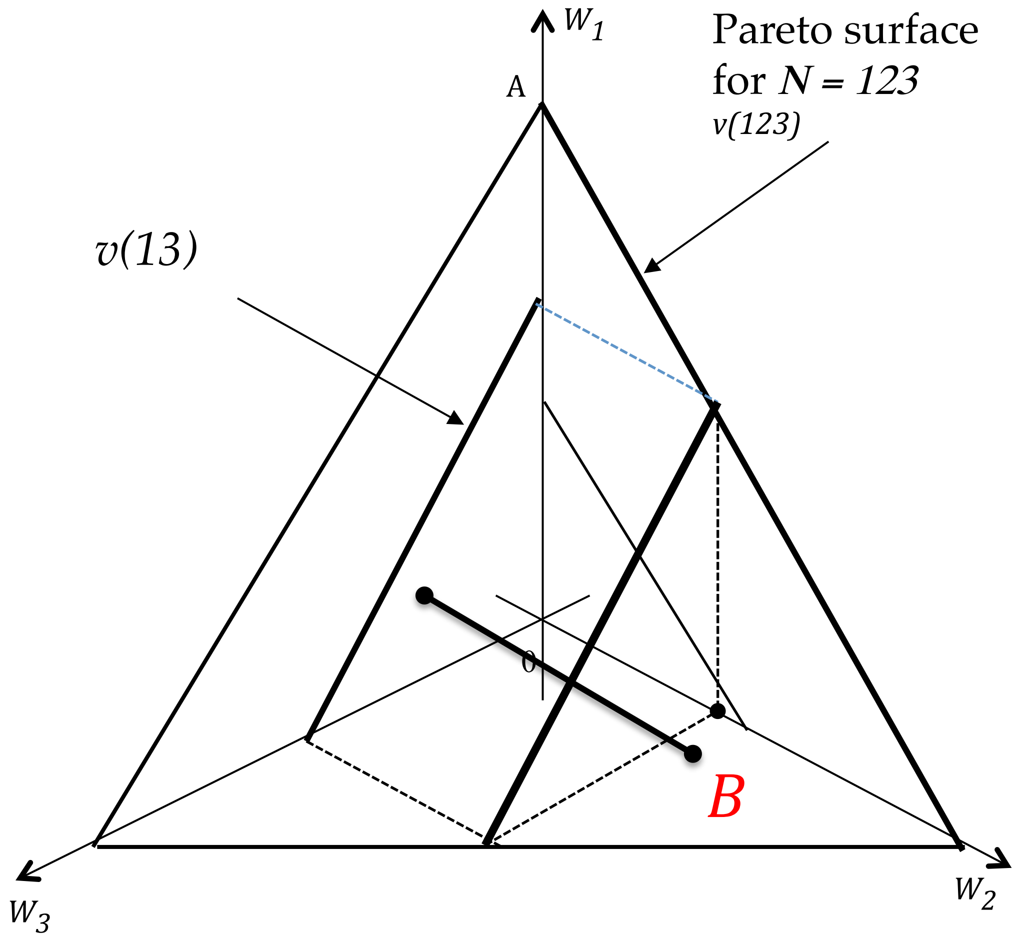
Do again the projection exercise, that is:

Projection of *v(13)*
on the Pareto surface
for N=123



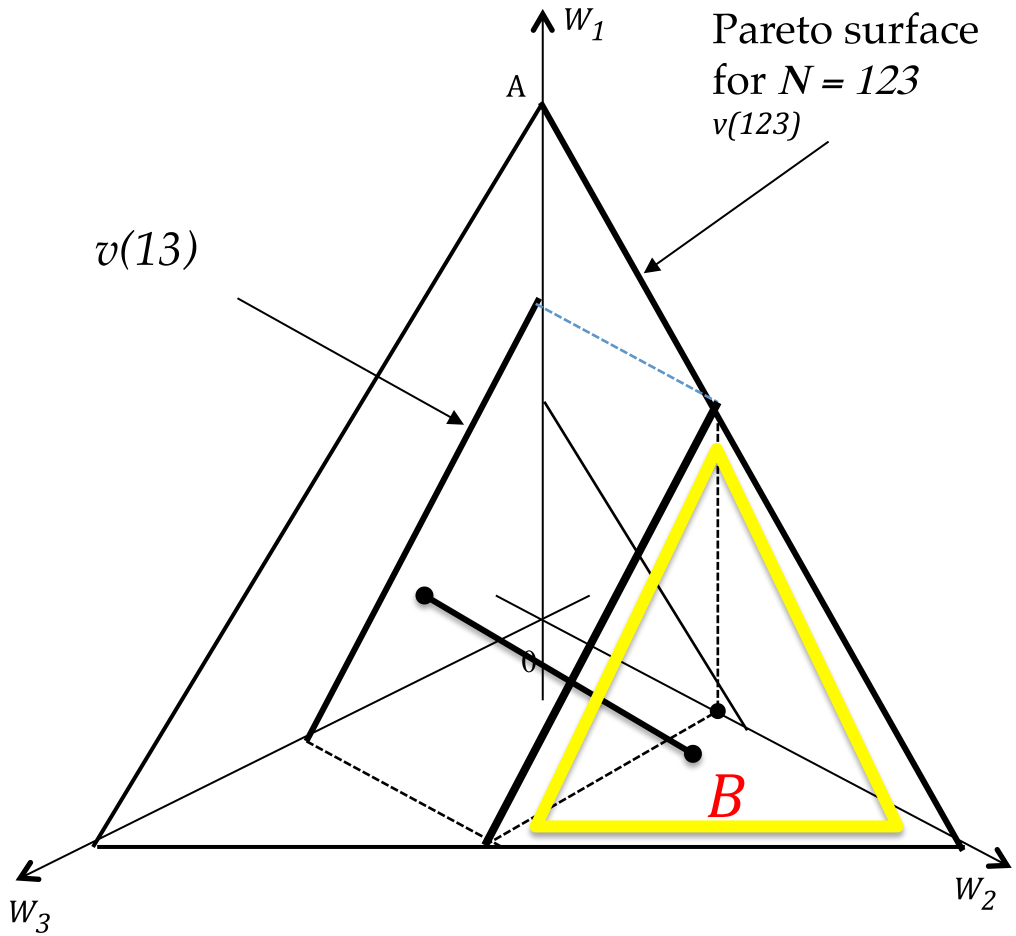
implication : «*B*», a Pareto efficient point for coalition (123),

is rejected (« blocked ») by coalition (13), because dominated by points of v(13)



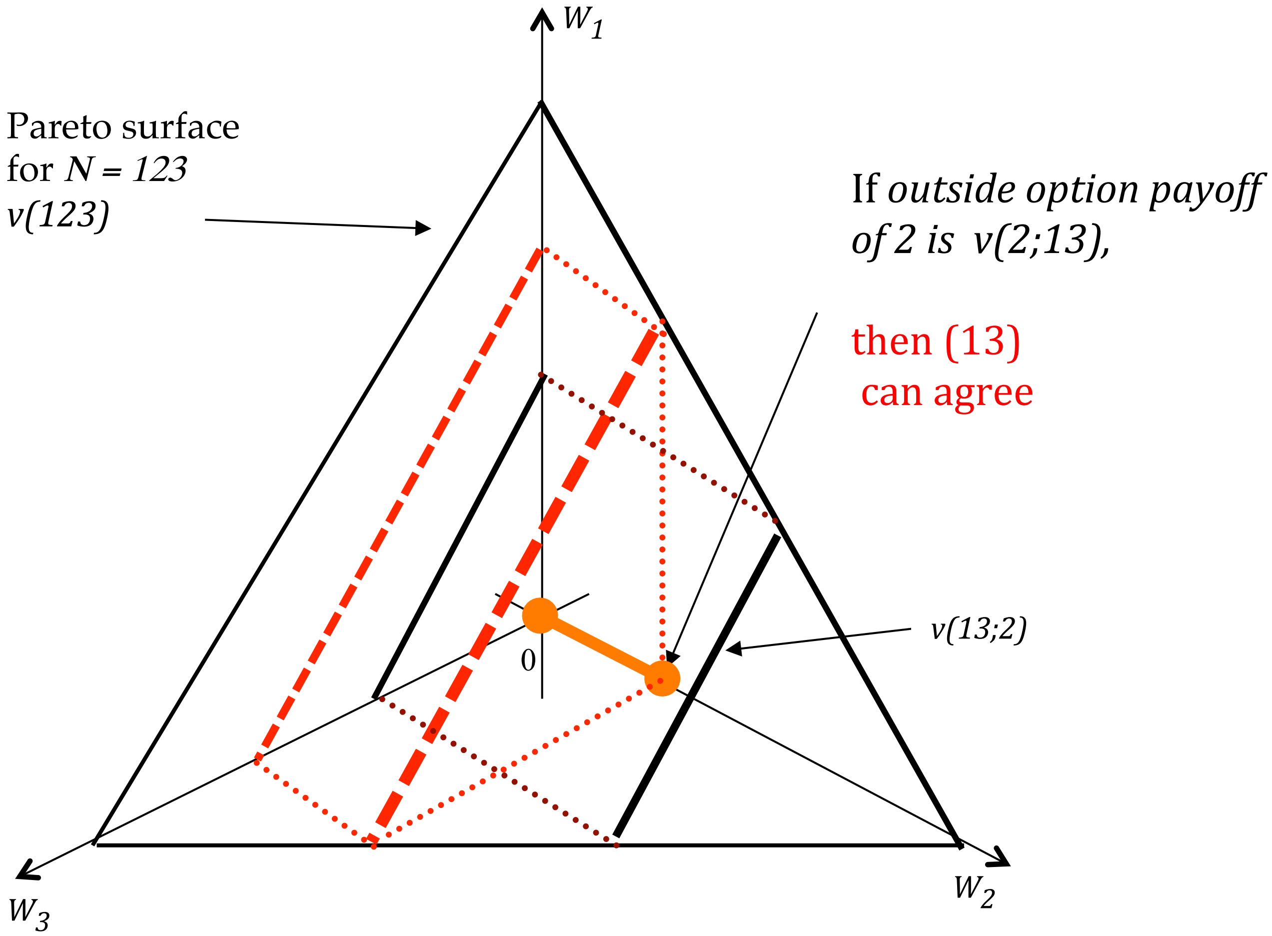
Thus, «*B*», a Pareto efficient point for coalition (123),

is rejected by coalition (13), as all points in a yellow zone, like on p. 9 above



3.2 *Introducing the externality from (13) on player 2,and possible defection of 2*

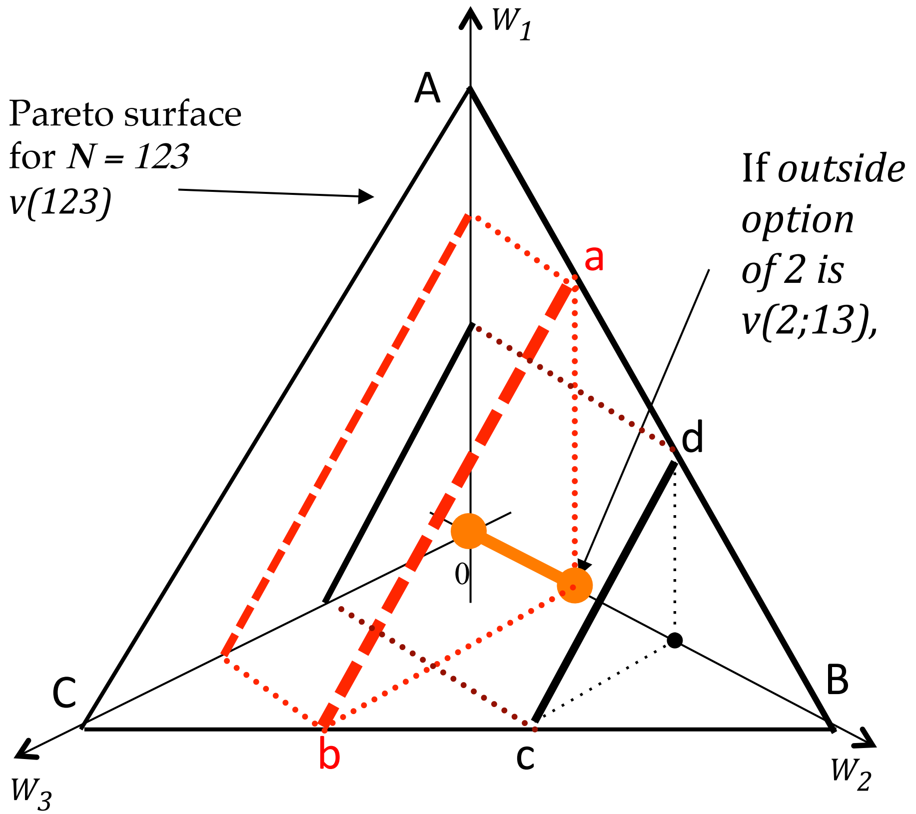
- **Recognize the externality generated by (13) on the payoff W_2 of player 2**
- **Locate 2's outside option payoff**
- **Determine coalition (13)'s reaction.**



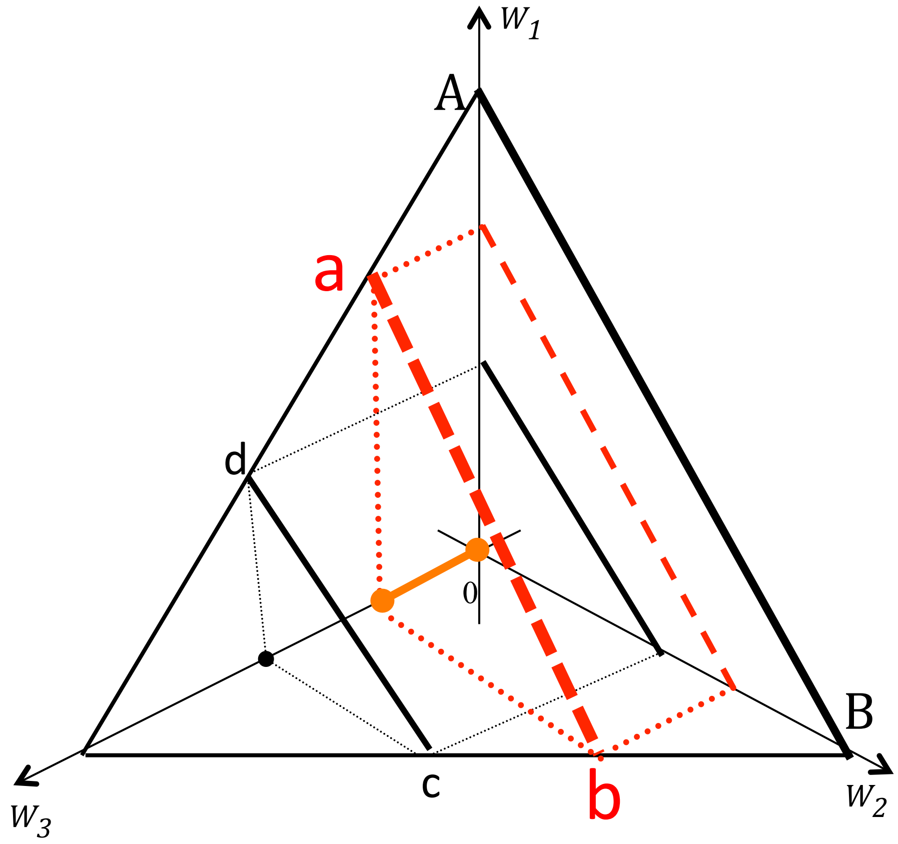
Thus,

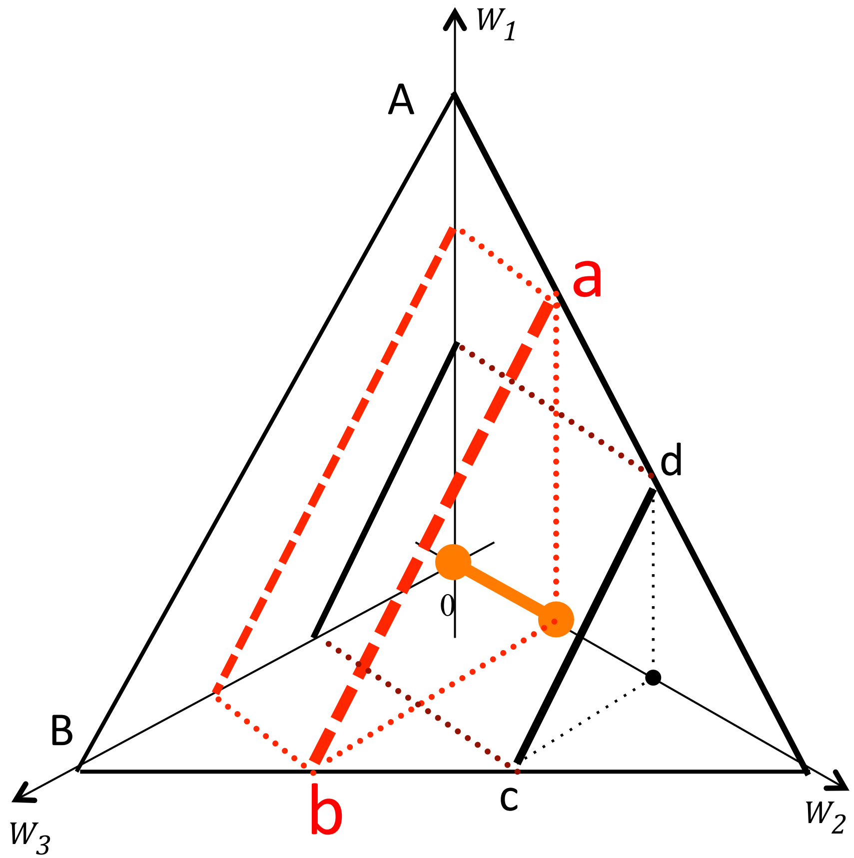
2's requirement of his outside option payoff ALSO BITES on the Pareto surface:

Of all Pareto efficient points available in the game, « AdcC »,
the requirement excludes the area « AabC » for reason of outside option of 3
and leaves only the area « abcd » for sharing v(123)



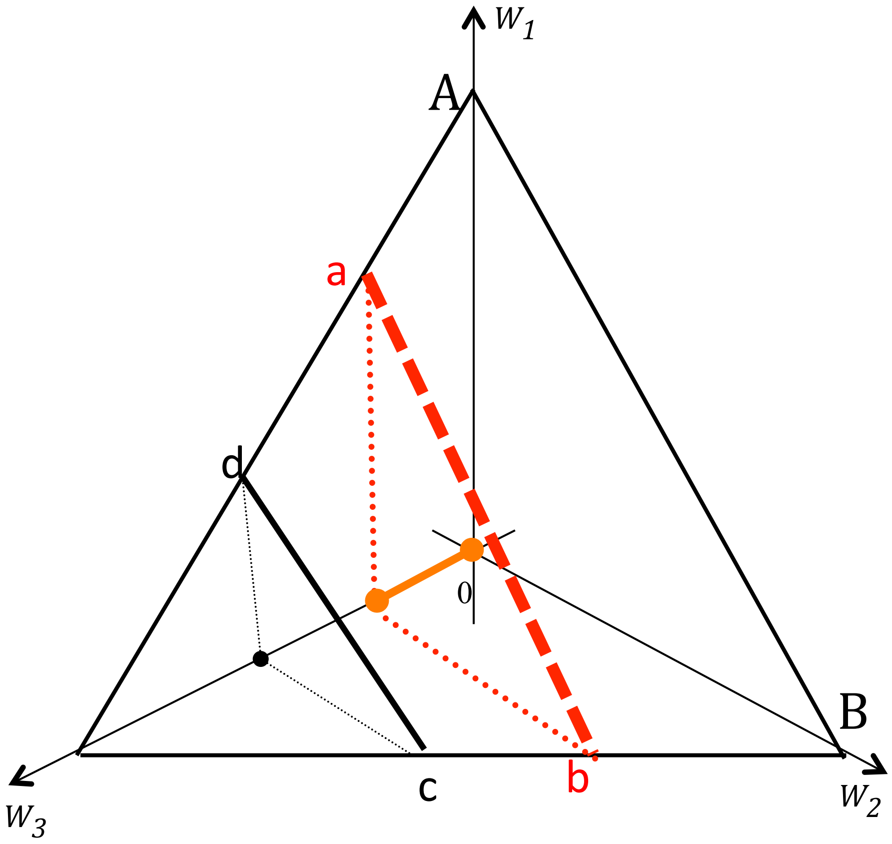
Thus, recognizing the outside options of both 3 and and 2 leads to two « bites » ‘above’ ab :

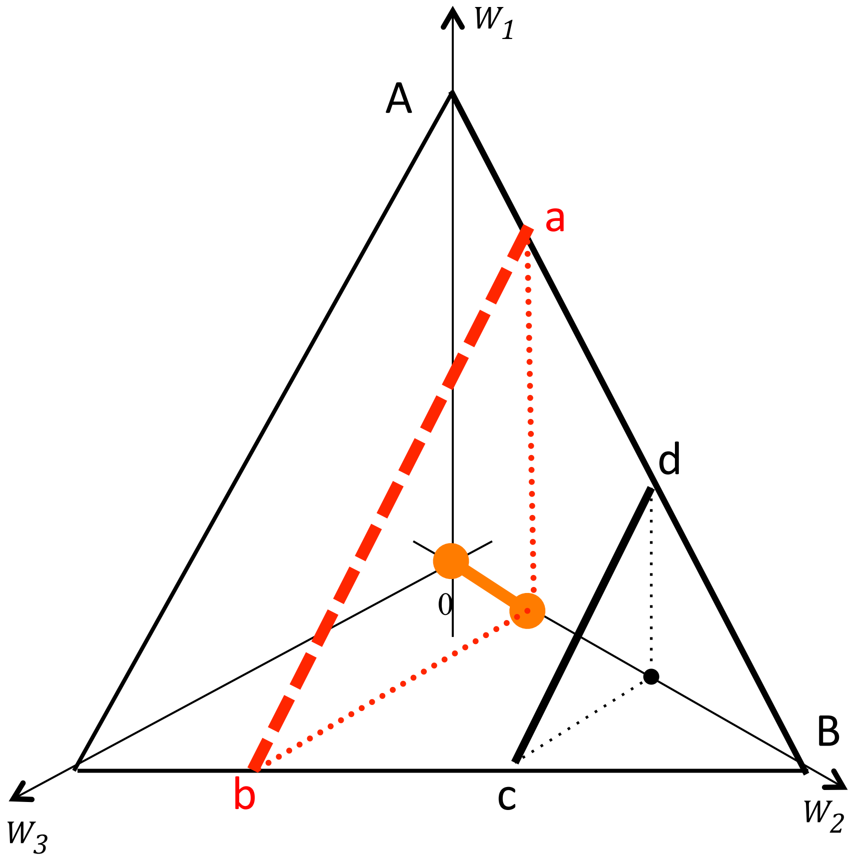


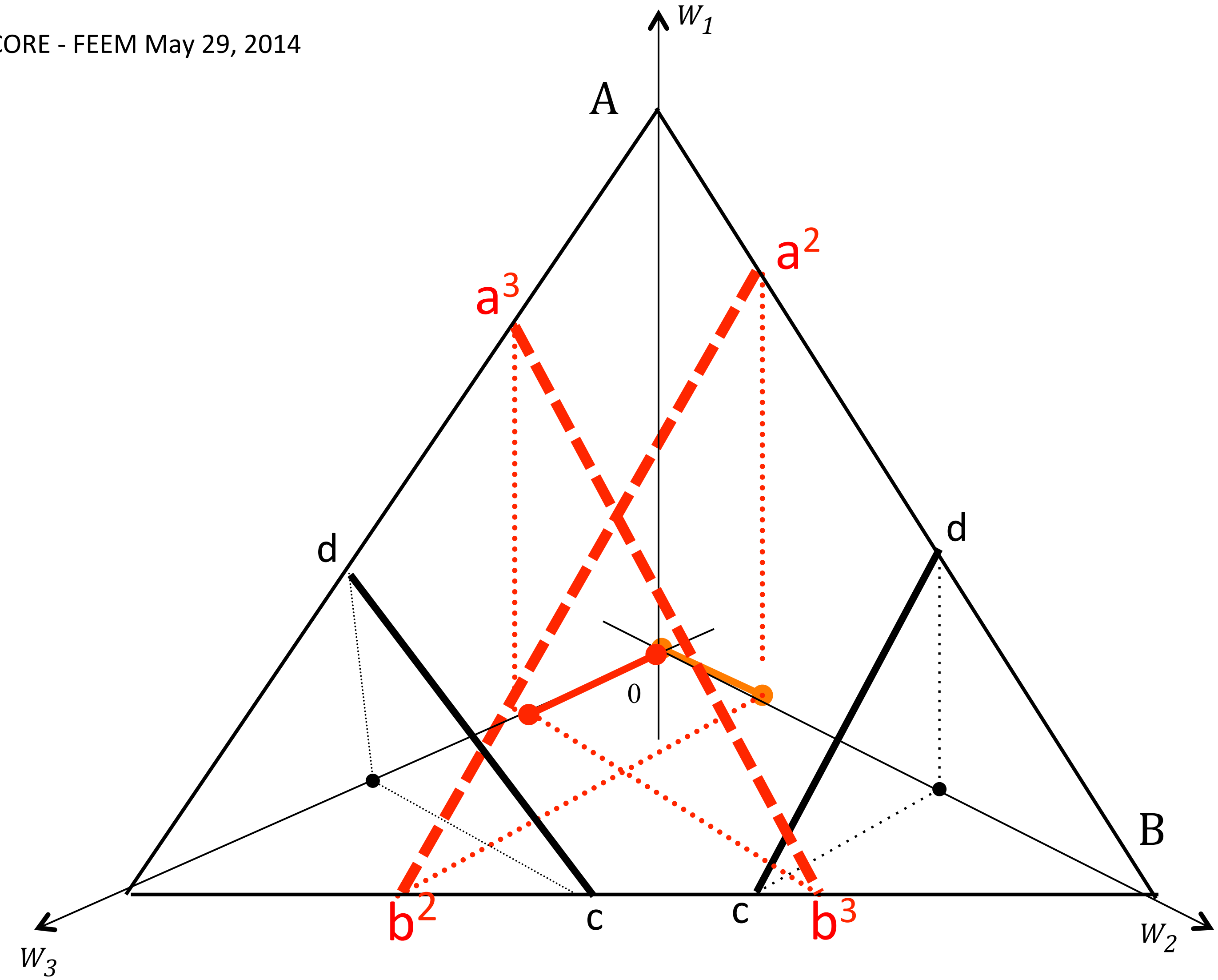


Putting the two together (not showing the projections anymore), we must go

from this pair : to one diagram





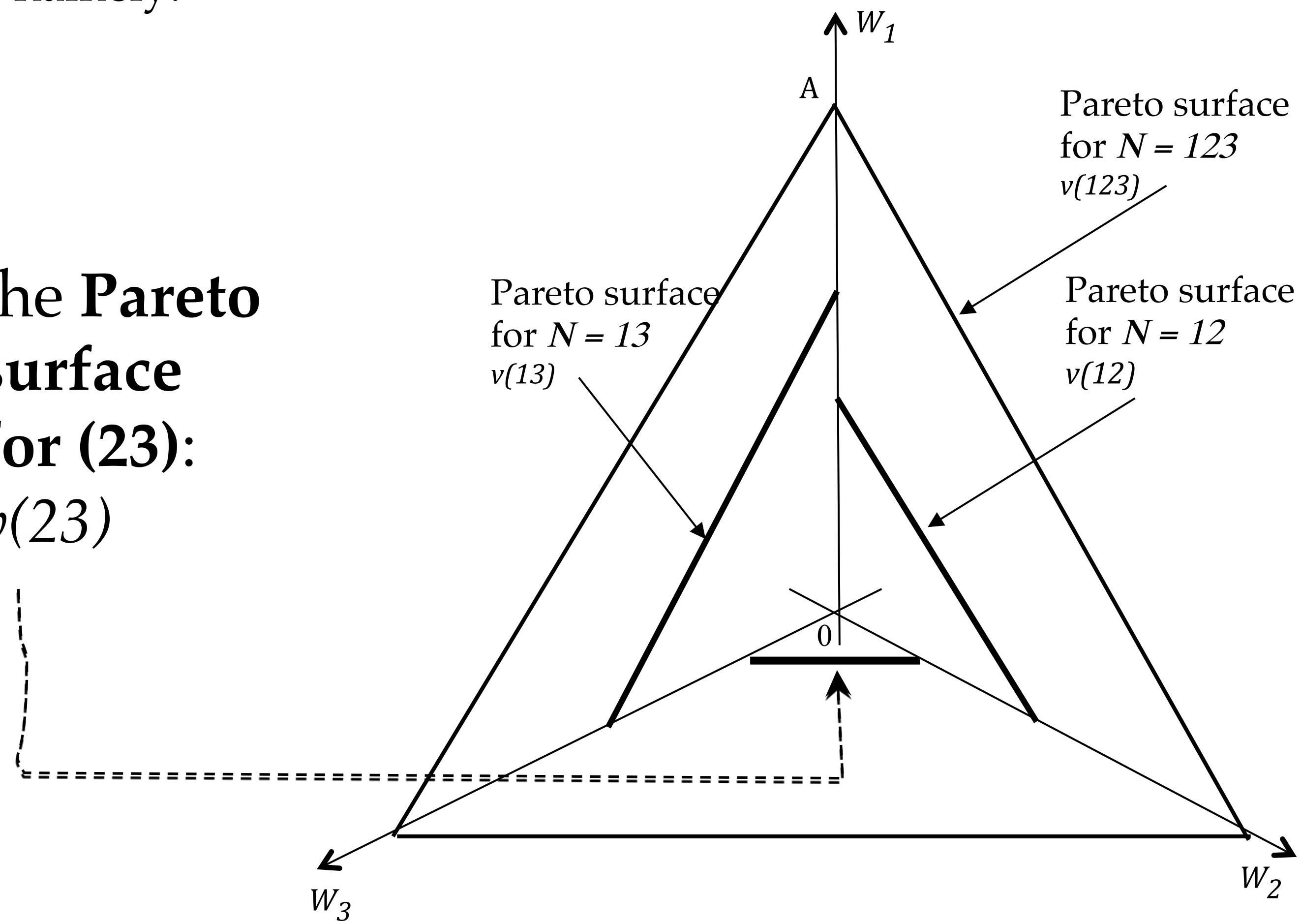


Both 2′s and 3′s requests for their outside option payoffs do « bite » on the Pareto surface.
Only for Pareto efficient points lying ***in the intersection*** of the two areas abcd can coalition (123) be internally stable *for players 2 and 3*. How about player 1?

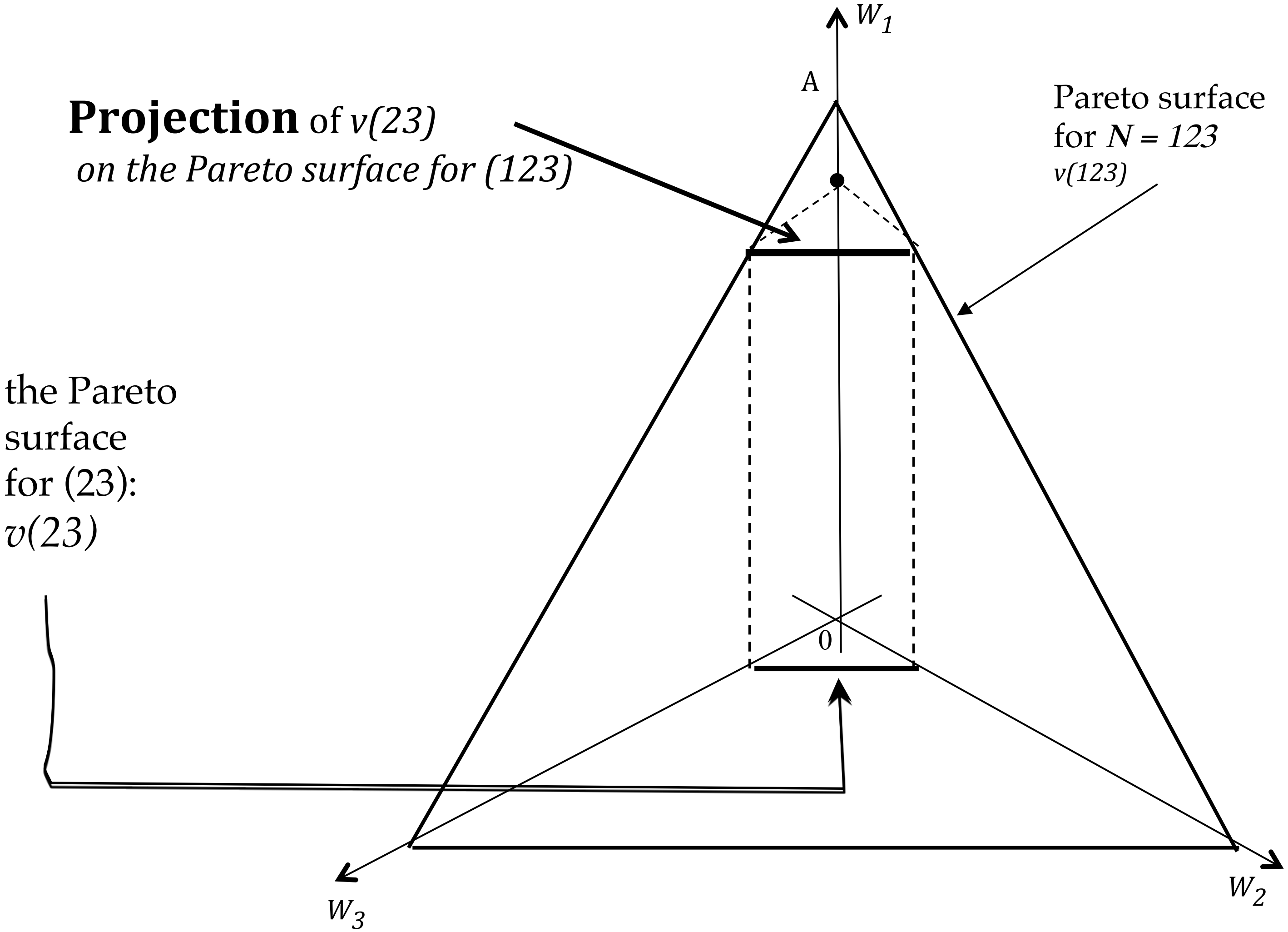
3.1 Introducing the third player - Preliminaries

Start all over again with the same game N= {1,2,3}
and consider now what coalition (23) can do,
namely:

the **Pareto surface for (23)**:
v(23)

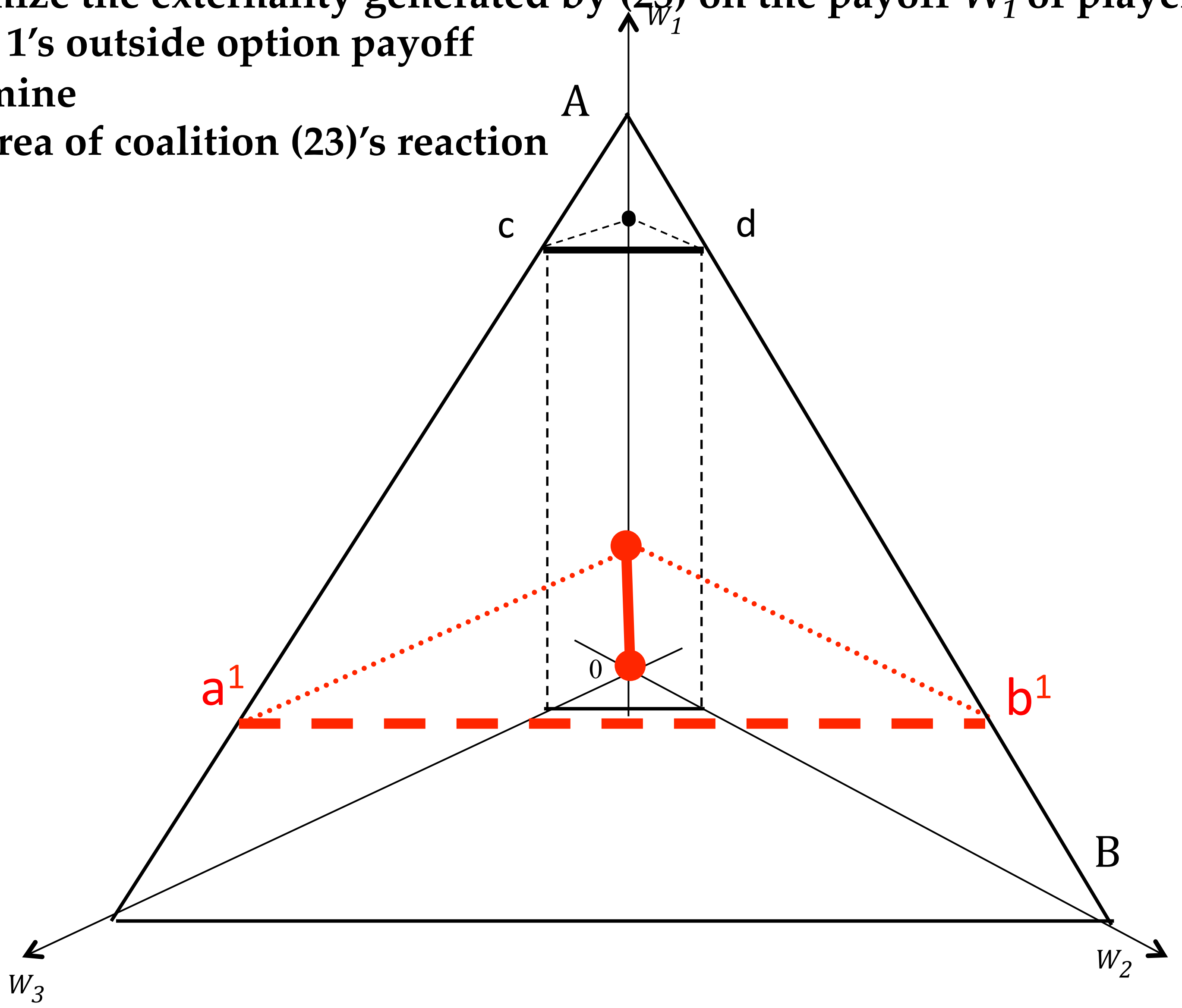


Do again the projection exercise :

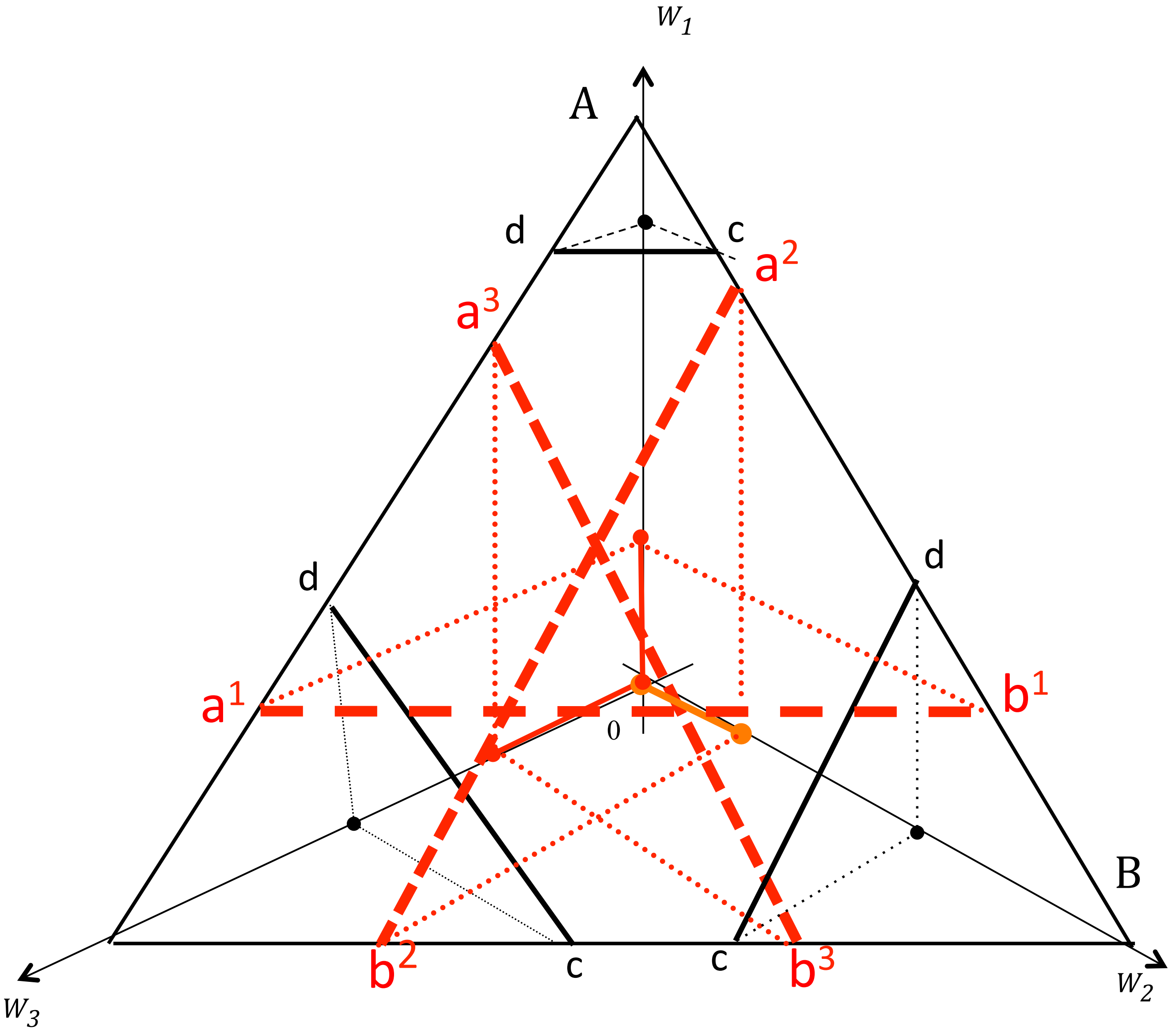


3.2 Introduce the externality from (23) on player 1,and possible defection of 1,

- **Recognize the externality generated by (23) on the payoff W_1 of player 1**
- **Locate 1's outside option payoff**
- **Determine**
  the area of coalition (23)'s reaction



3.3 *Plugging this in the diagram of p. 28, one gets for the game :*



And more precisely: …

Look at the **intersection** of the three areas:

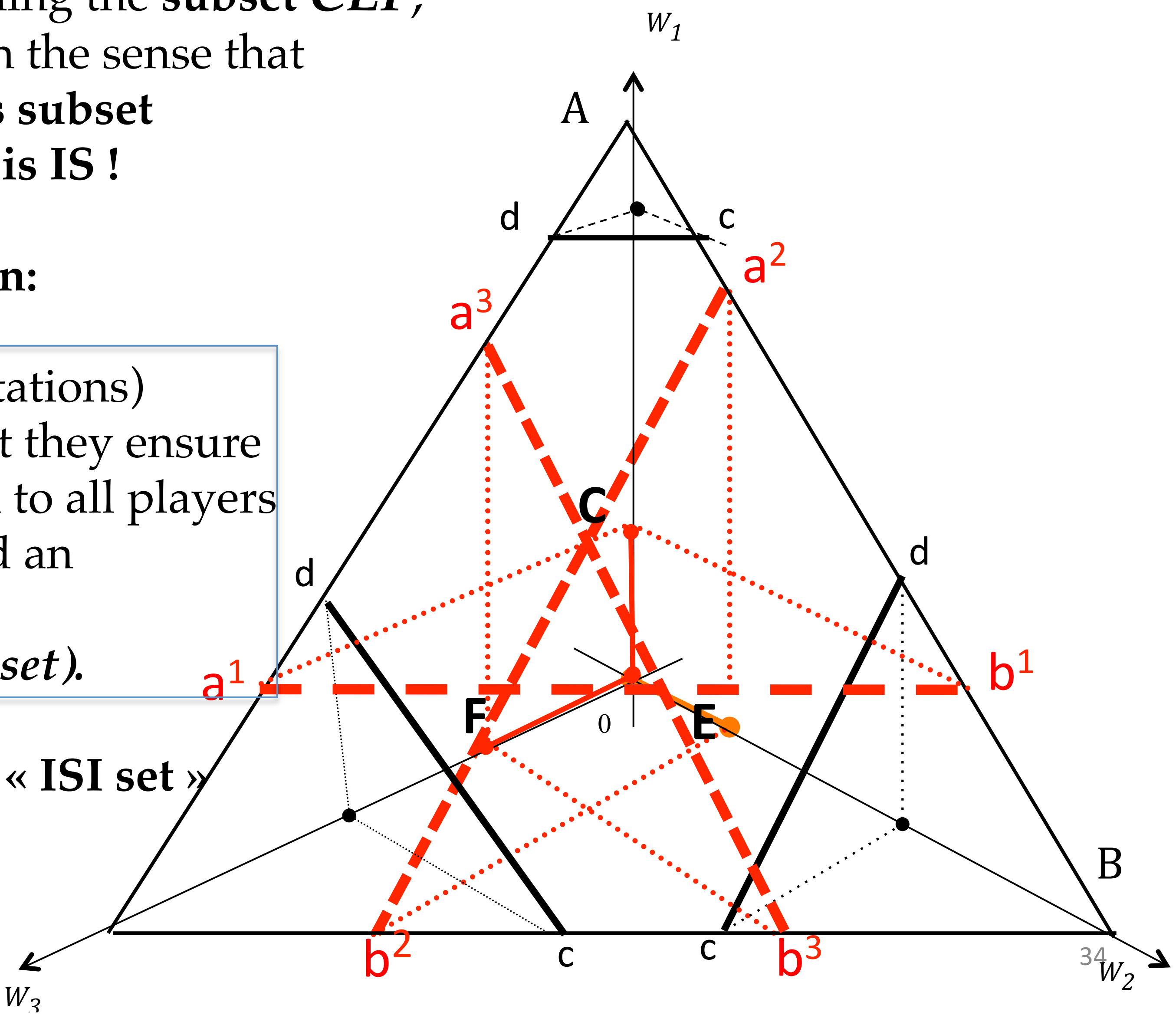
- $a^3 b^3$ cd, required by player's 3; - $a^2 b^2$ cd, required by player's 2; - $a^1 b^1$ cd, required by player's 1.

As they overlap, forming the **subset *CEF*,** they are compatible in the sense that **any point within this subset of the Pareto surface is IS !**

Formally: Definition:

A set of points (imputations) with the property that they ensure their free rider option to all players of a coalition is called an ***Internal Stability Imputations set (ISI set).***

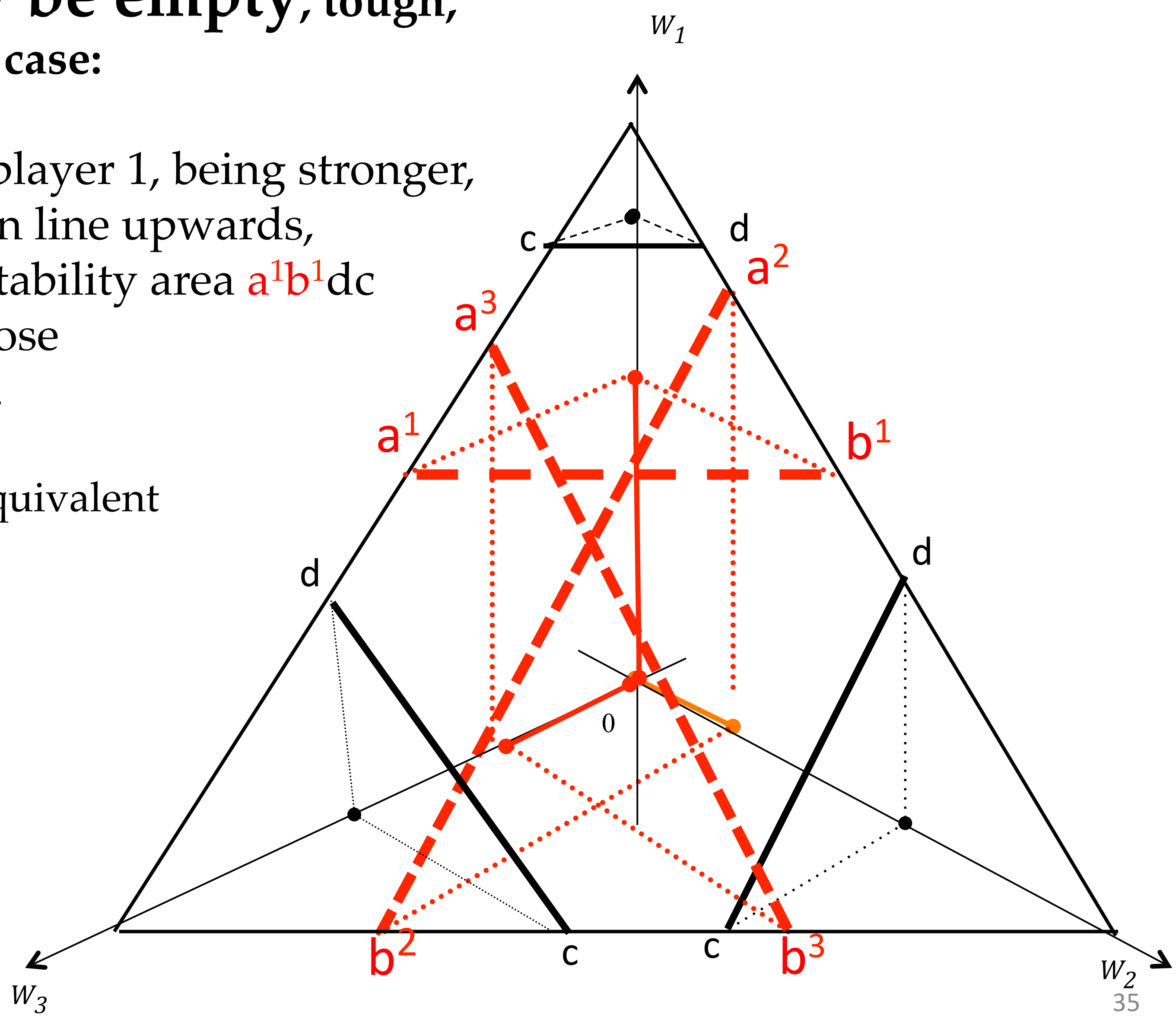
In the Fig., ***CEF*** **= the « ISI set »**



The ISI set may be empty, tough, as in the following case:

here the request of player 1, being stronger, shifts the a^1b^1 broken line upwards, rendering his acceptability area a^1b^1dc *incompatible* with those of the other players.

(the area *apparently* equivalent to CEF is empty)



4. The « PISI point »

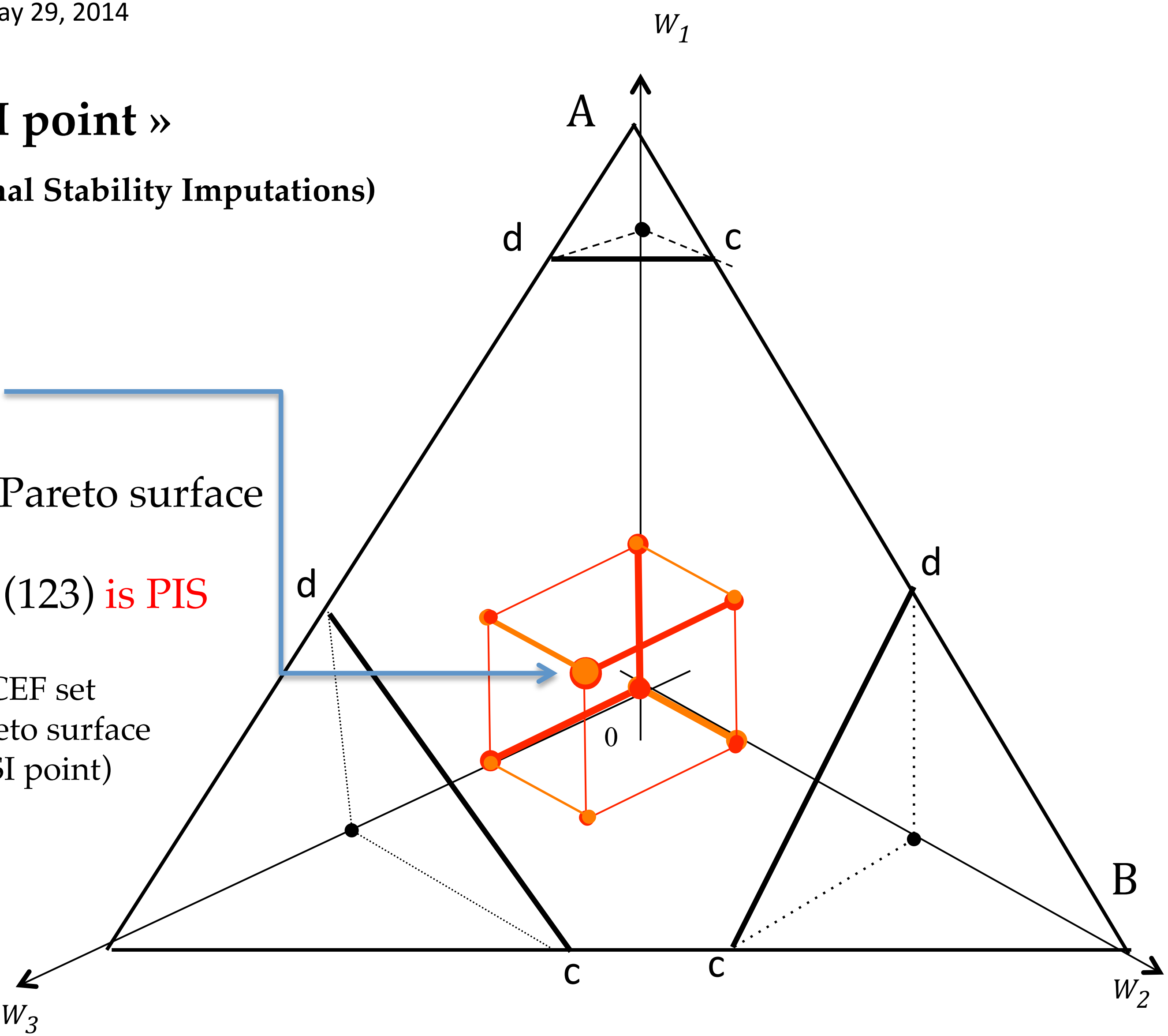
(for Potential Internal Stability Imputations)

If the PISI point

lies « below » the Pareto surface

then the coalition (123) is PIS

(the equivalent to the CEF set
is the subset of the Pareto surface
that dominates the PISI point)



5. Results thus obtained.

For the environmental externality game, we have, thus far:

1. Proposed that a free rider's « sensitivity to the externality » emitted by the other coalition members be **measured** along the axis of his payoff.

2. Illustrated the basic notion of « outside option payoff » of a free rider, on the Pareto surface of the coalition he is a member of, in terms of a **broken line** ab across that surface.

3. Identified with this broken line an **area** abcd within the Pareto surface which is the set of imputations that the member of the coalition can claim without endangering the stability of the coalition.

4. After having repeated this construction for **all** members of a coalition, have defined as the *intersection* of the said areas the « Internally Stable Imputations Set » (**the ISI set**) of the coalition. That set may be empty.

5. A similar construction can be made to construct a set of « Potentially Internally Stable Imputations (**the PISI set**). That set may be empty, but it may be non empty even if the ISI set is empty.

Analytic formulation of the above:

Feasibility conditions of internal stanility,
for any coaolition with $n \geq 3$ players:

- for the outside option payoff of a player, say *i*,
  to be acceptable by $N \setminus \{i\}$: $v(i; N \setminus \{i\}) + v(N \setminus \{i\};i) \leq v(123)$

- For the « ISI set » to be non empty:
  that the condition $v(i; N \setminus \{i\}) + v(N \setminus \{i\};i) \leq v(N)$ holds for every i €N

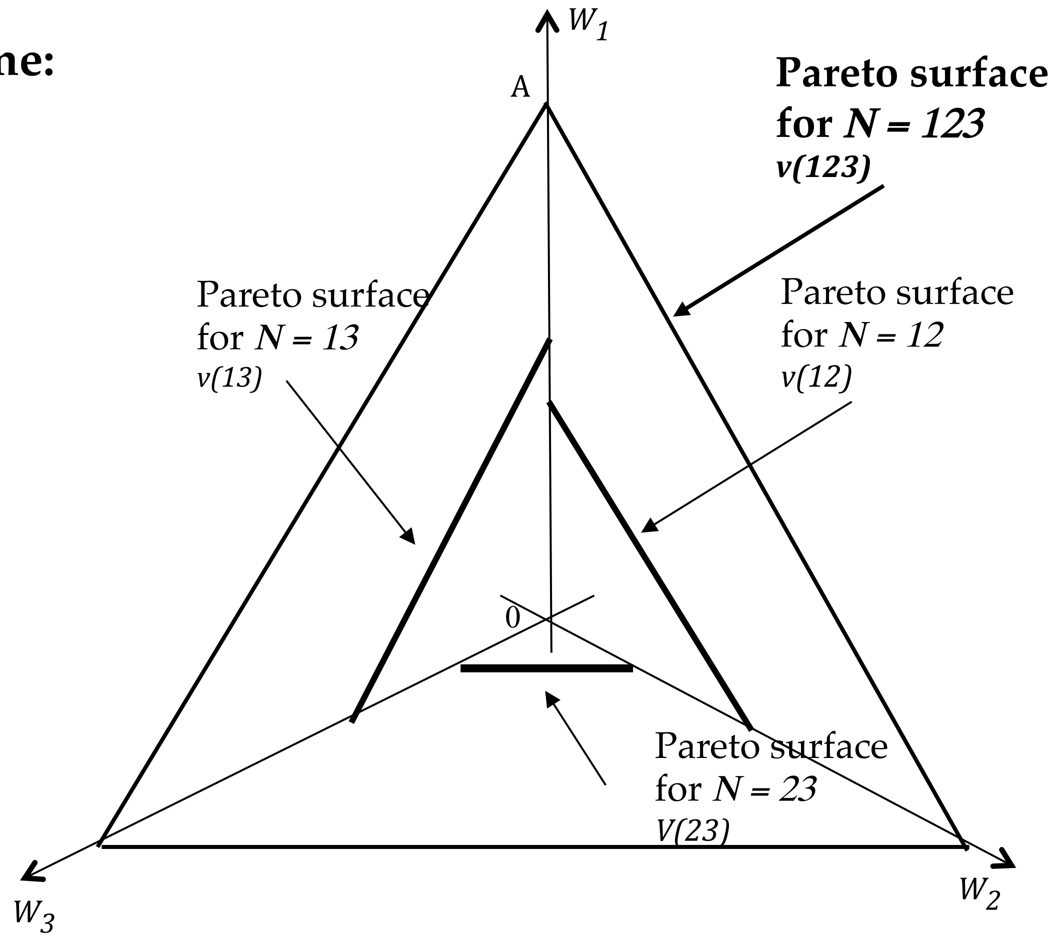
- For the « PISI set » to be non empty:
  that $v(i; N \setminus \{i\}) + v(N \setminus \{i\};i) \leq v(N)$ holds for every i €N

6. Comparing with a « core stable » coalition

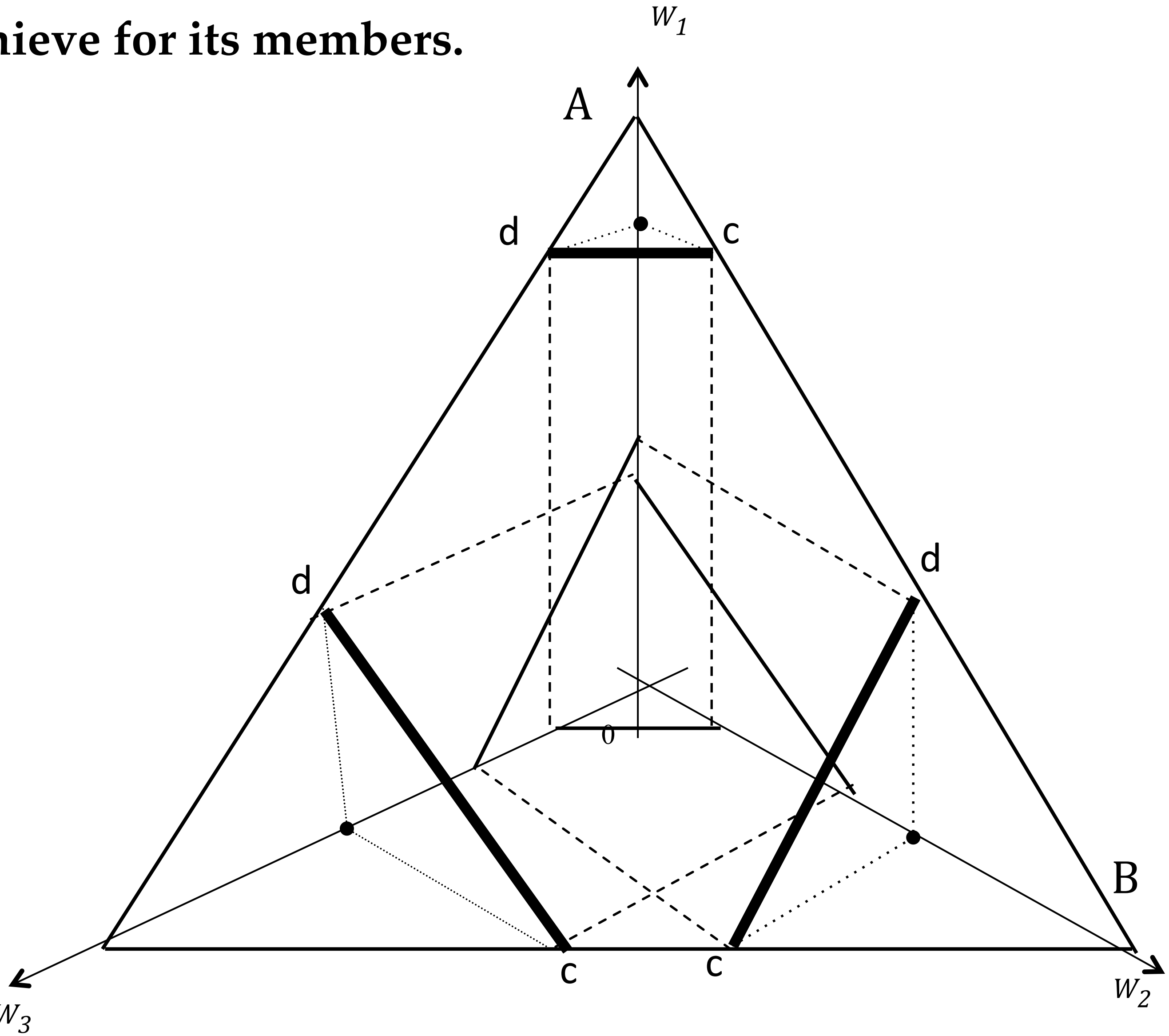
Graphical representation of the core of a 3 player game

See Mas Colell, Whinston and Green 1995, chap 18, Appendix A

Data of the game:



Do the projections exercise on the Pareto surface of what each coalition can achieve for its members.



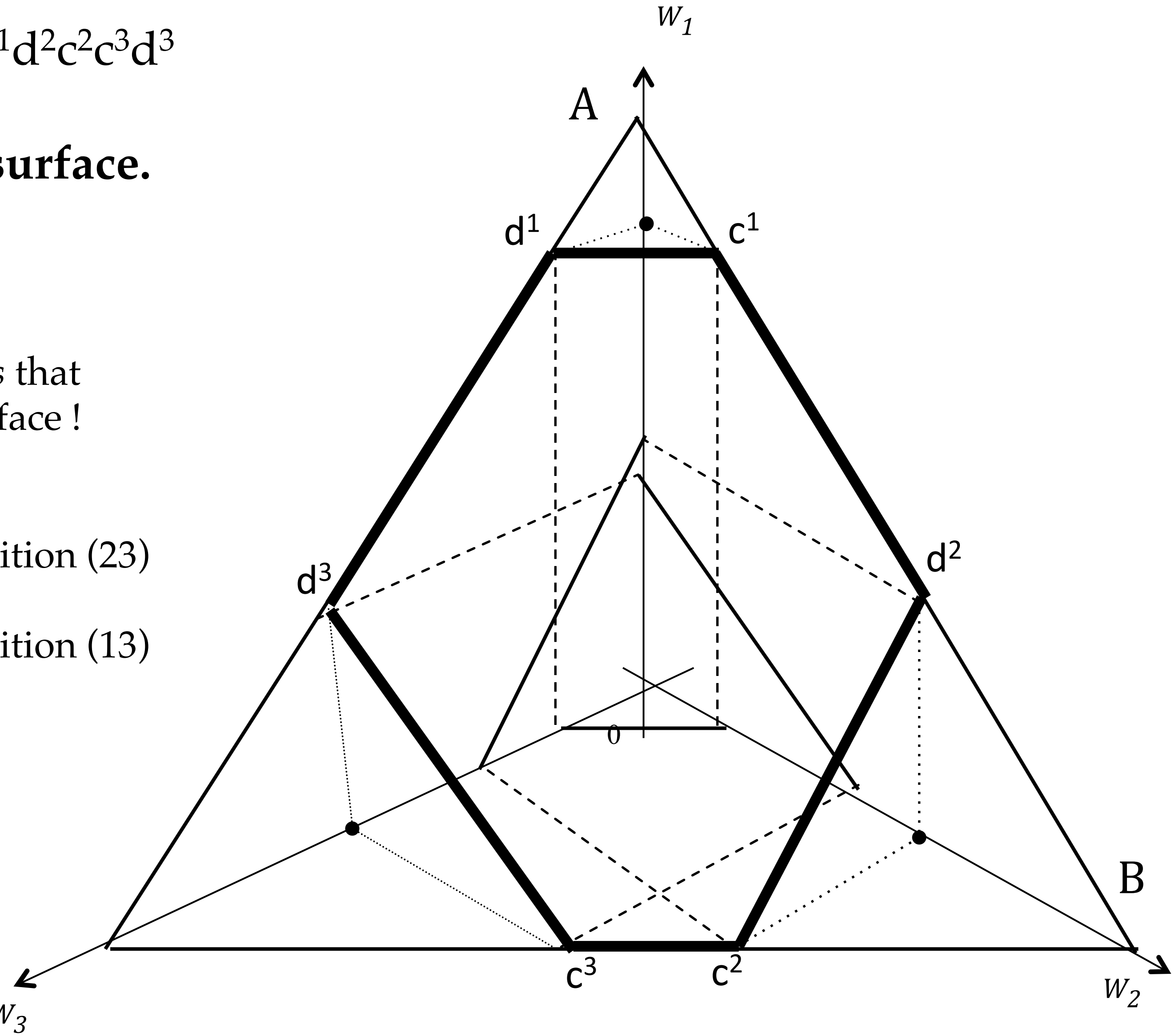
Then…

The (γ-)CORE of the game is

the subset $d^1c^1d^2c^2c^3d^3$

of the Pareto surface.

Notice:
here, it is *coalitions* that
« bite » on the surface !
Not individuals.
e.g.:
the « bite » of coalition (23)
is area d^1c^1A
the « bite » of coalition (13)
is area d^2c^2B

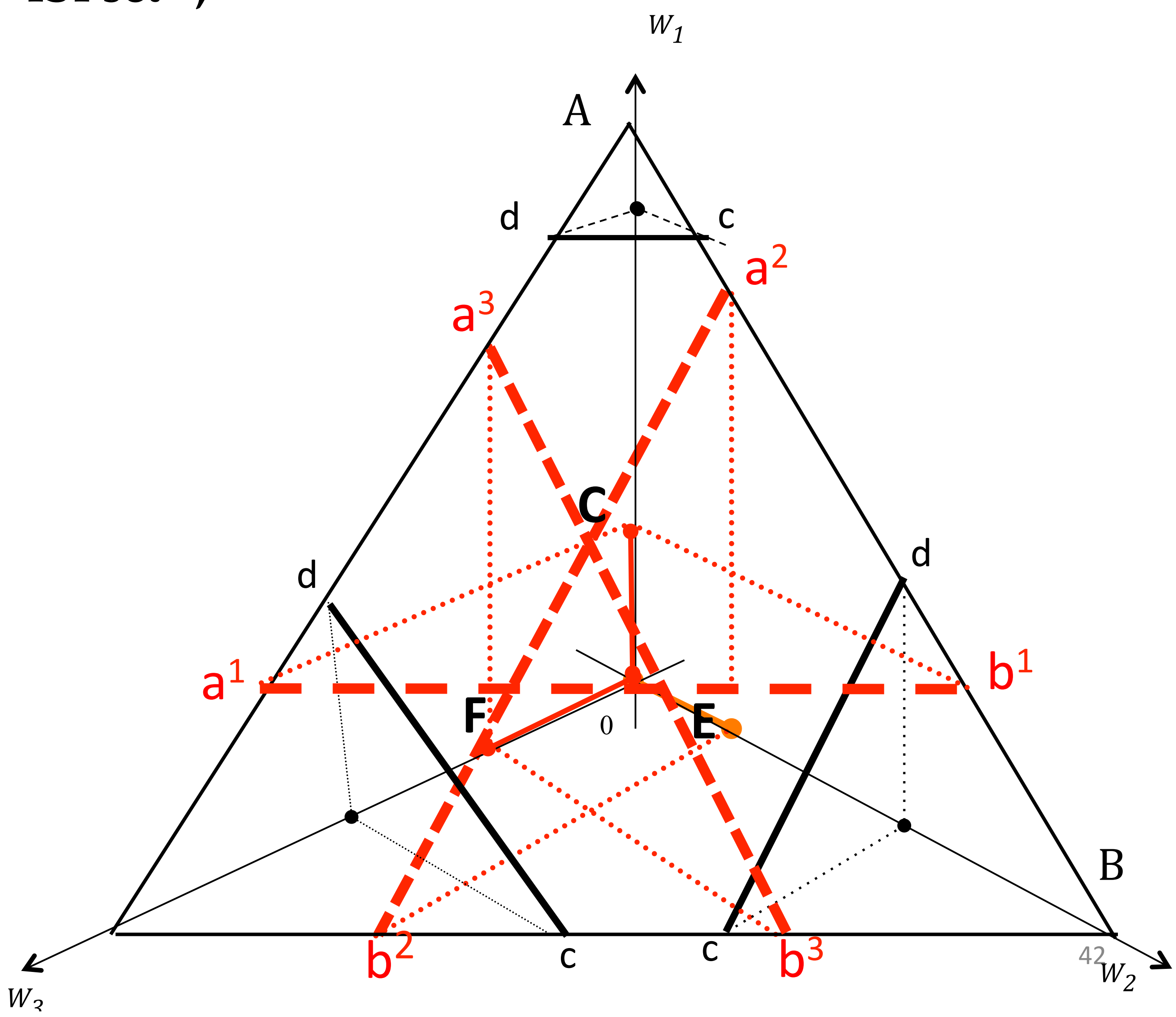


To compare *Internal Stability* with *Core Stability*,

let us go back to the « ISI set »,

as defined earlier,

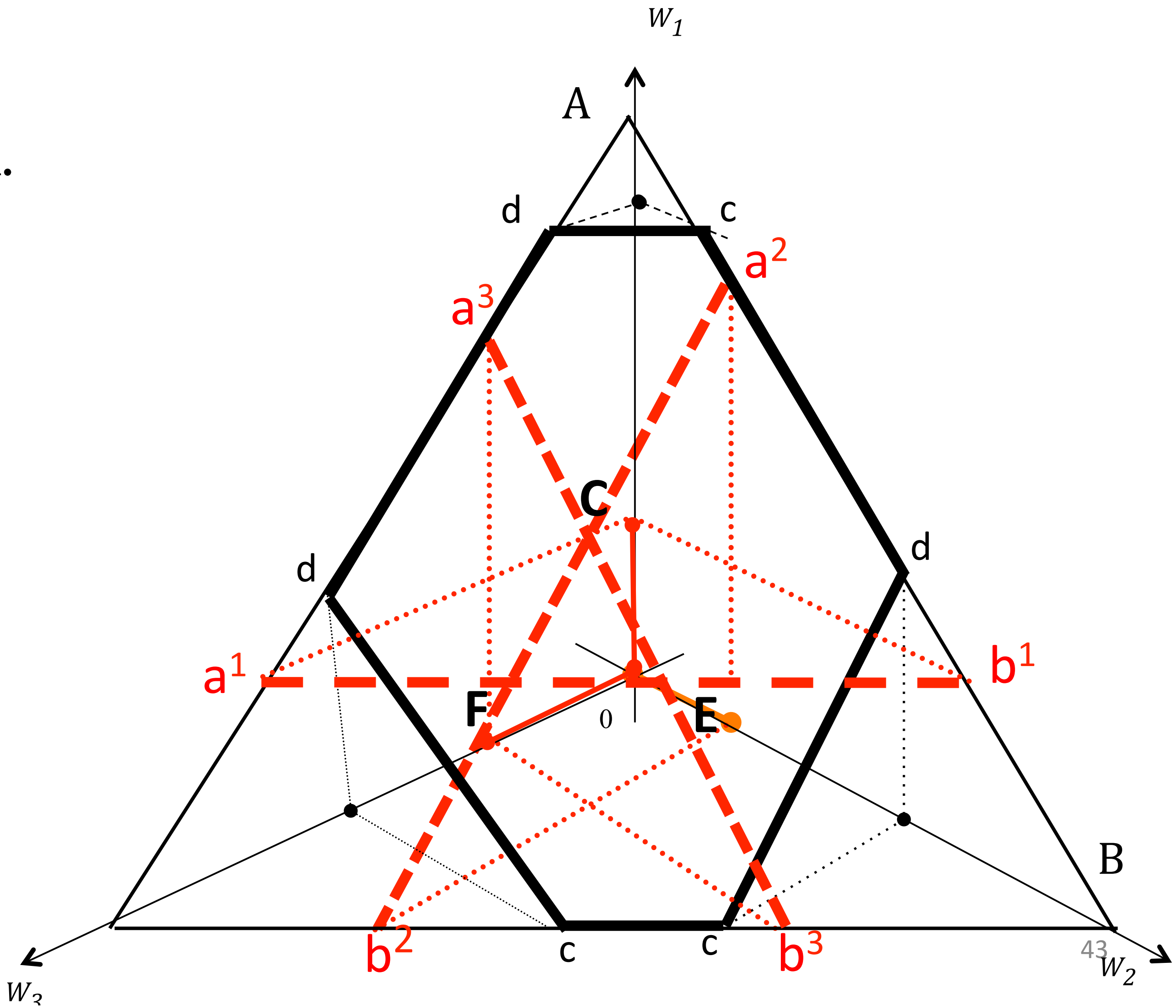
namely CEF,



To compare Internal Stability with Core stability,

and draw the core

in that same diagram.

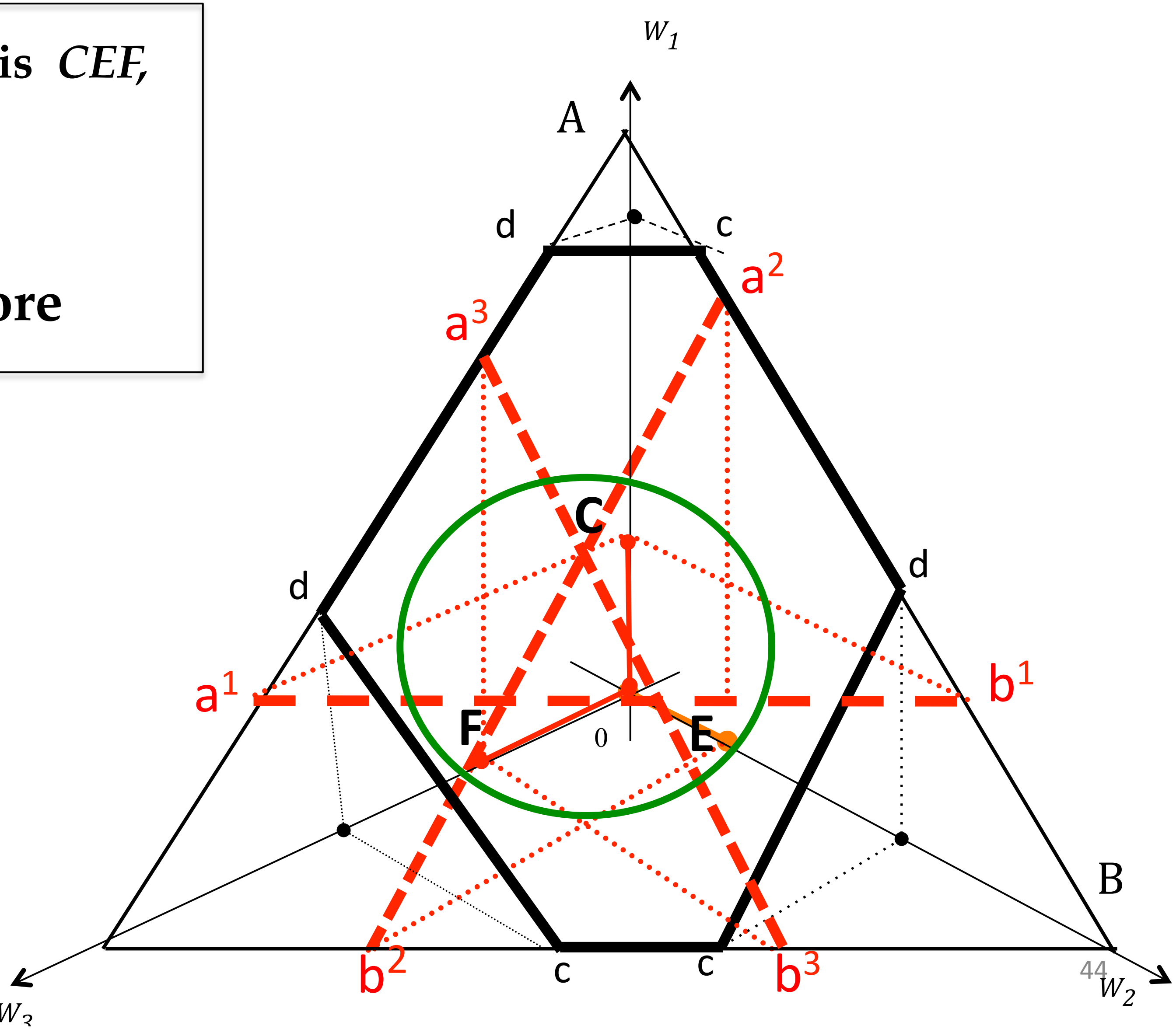


Main proposition of this paper :

the « ISI set », which is *CEF*,

is

a subset of the core



Corollaries:

1. **If the externality vanishes for all players, the ISI set coincides with the core**

2. **If the core is empty, so is the ISI set**

3. **If the ISI set is empty, it does not imply that the core be empty**

4. **If the core consists of one point only,**
   the ISI set (if it exists) can consist of only *one particular configuration* of the players' sensitivity to the externality.

Concluding remarks

1. On TU vs NTU

same graphical methodology (diagrams harder to draw!)

Don't see why NTU would prevent reformulating the definitions introduced here, and imply different conclusions

2. On the two stability notions: IS (PIS) vs CS

In IS and PIS, the source of stability lies in the other players' resources, used to deter free riding.

In CS the source of stability rests in the own resources of each coalition, used to consider alternatives.

3. On **methodology**: Ultra simple models often give insights that may usefully inspire large ones !

THANKS FOR YOUR ATTENTION