STOCHASTIC CAPACITY EXPANSION MODELS :

Risk exposure and Good-deal valuation

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Electricity markets : Investment models

Generation capacity expansion models have a long tradition in the power industry
- Models that optimally select the timing and the level of investment over a time horizon

▶ Well adapted for the regulated monopoly industry (from the 50ties in EdF)
  - Risk was largely passed to the consumer through average cost pricing

▶ Still very popular after the restructuring of the sector
  - Interpretable as equilibrium in a competitive environment
  - What about the risk ?

Those models have been adapted to stochastic programming in order to accommodate wide ranges of uncertainties
- Estimation of the margin profit distribution of power plants
- Risk averse formulation using the good-deal risk measure
Outline

Generation Capacity Expansion Models

1. Introduction: optimization, equilibrium and risk
2. Estimation of the distribution of the margin profit
3. Risk averse formulation: good-deal risk measure
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A basic capacity expansion model

- Two stages: invest in $t = 0$, operate in $t = 1$
- Price insensitive demand
- Invest $x(k)$ in K technologies (coal, CCGT, ...)
- Operate them at $y(k, \ell)$ in time segment $\ell$
- Satisfy or curtail the demand $d(\ell)$

A deterministic optimization problem:

$$\min_{x \geq 0} \sum_{k=1}^{K} I(k)x(k) + Q(x)$$

$$Q(x) = \min_{y, z} \sum_{\ell=1}^{L} \tau(\ell) \left( \sum_{k=1}^{K} c(k)y(k, \ell) + PCz(\ell) \right)$$

s.t.

$$0 \leq y(k, \ell) \leq x(k)$$

$$0 \leq \sum_{k=1}^{K} y(k, \ell) + z(\ell) - d(\ell)$$
Parameters and units

\( I(k) \)  Investment cost (annuity) of technology \( k \) [EUR/MWyear]
\( c(k) \)  Operating cost of technology \( k \) [EUR/MWh]
\( PC \)  Value of Loss Load or price cap [EUR/MWh]
\( d(\ell) \)  Demand level [MW]
\( \tau(\ell) \)  Number of hours in time segment \( \ell \)

\( \pi(\ell) \)  Electricity price in time segment \( \ell \) [EUR/MWh]
\( \mu(k, \ell) \)  Gross margin of technology \( k \) in time segment \( \ell \) [EUR/MWh]
From optimization to equilibrium

Equivalent to a perfect competitive equilibrium in an economy where

\[ \text{N agents maximize their profit } (\nu = 1, \ldots; N - \text{ given } \pi(\ell)) : \]

\[
\max_{x_{\nu} \geq 0} \sum_{k=1}^{K} \sum_{\ell=1}^{L} \tau(\ell) \left( \pi(\ell) - c(k) \right) y_{\nu}(k, \ell) - I(k)x_{\nu}(k) \\
\text{s.t. } 0 \leq y_{\nu}(k, \ell) \leq x_{\nu}(k)
\]

Market clearing conditions (with price cap) :

\[
0 \leq \pi(\ell) \perp \sum_{\nu=1}^{N} \sum_{k=1}^{K} y_{\nu}(k, \ell) + z(k, \ell) - d(\ell) \geq 0 \\
0 \leq z(\ell) \perp PC - \pi(\ell) \geq 0
\]

Investment criterion

\[
0 \leq x(k) \perp I(k) - \sum_{\ell=1}^{L} \tau(\ell) \mu(k, \ell) \geq 0
\]
Introducing risk

- Uncertainties: demand, operating costs, but also regulatory risk
- Invest in $t = 0$ before the actual realization of parameters
- $(\Omega, P)$ a probability space where:
  - Scenario $\omega \in \Omega$: probability $p(\omega)$ and corresponding realization of:
    
    $$(c(k, \omega), d(\ell, \omega), PC(\omega))$$

A stochastic optimization problem:

$$\min_{x \geq 0} \sum_{k=1}^{K} I(k) x(k) + \mathbb{E}_P [Q(x, \omega)]$$

$$Q(x, \omega) = \min_{y, z} \sum_{\ell=1}^{L} \tau(\ell) \left( \sum_{k=1}^{K} c(k, \omega) y(k, \ell, \omega) + PC(\omega) z(\ell, \omega) \right)$$

s.t.

$$0 \leq y(k, \ell, \omega) \leq x(k) \quad \tau(\ell) \mu(k, \ell, \omega)$$

$$0 \leq \sum_{k=1}^{K} y(k, \ell, \omega) + z(\ell, \omega) - d(\ell, \omega) \quad \tau(\ell) \pi(\ell, \omega)$$
Stochastic equilibrium

- Also amenable to a perfectly competitive stochastic equilibrium
- The dual variables \((\pi(\ell, \omega), \mu(k, \ell, \omega))\) are now contingent on \(\omega\)
- The investment criterion becomes a NPV

\[
0 \leq x(k) \perp I(k) - \mathbb{E}_P \left[ \sum_{\ell=1}^{L} \tau(\ell) \mu(k, \ell, \omega) \right] \geq 0
\]

- Interpretation:
  - Agents are risk neutral / there exist Arrow-Debreu securities (not represented explicitly)
  - All agents in the economy value their investment according to a common exogenous discount factor (CAPM)
Restrictions and extensions

Extensions:

- Multi-period problem
- Price sensitive demand (welfare optimization)
- Storage possibilities, electric grid,...
- Emissions constraints (CO$_2$, NO$_x$,...)

Restrictions: Some feature are amenable to optimization but not to equilibrium

- Unit commitment characteristics (MIP)
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work done with Professor A. Shapiro, Georgia Tech.
Motivations:

- Investors want to have the best estimation of the distribution of the gross margin
  ▶ compute statistics, measuring the risk (VaR), evaluating hedging strategies,...

- When the risk factors have continuous distribution, the solution is obtained by sampling methods

Sample Average Approximation:

\[ \tilde{\theta}^N \equiv \min_{x \geq 0} \sum_{k=1}^{K} I(k)x(k) + \frac{1}{N} \sum_{j=1}^{N} Q(x, \omega_j) \]

▶ The sample size is limited: one wants to refine it for the profit distribution
Nonanticipativity constraints

A Stochastic Program can be formulated as

$$\min_{x \in \mathcal{L}} \mathbb{E}\left[ \sum_{k=1}^{K} I(k)x(k, \omega) + Q(x(\omega), \omega) \right]$$

where $\mathcal{L}$ is the set of nonanticipativity constraints

$$\mathcal{L} := \{ x \in \mathcal{X} : x(\omega) = x \}$$

The associated Lagrange multipliers:

$$\lambda(k, \omega) = \sum_{\ell=1}^{L} \tau(\ell) \mu(k, \ell, \omega) - I(k)$$

$\lambda(k, \omega)$ is net margin of technology $k$ for a given realization of $\omega$
Estimating the distribution

We propose a sampling method:

1. Solve the initial stochastic problem (SAA method - check quality)
   - Optimal investment $\Rightarrow \hat{x}(k)$

2. Sample again the parameters (large): $\omega_1, \omega_2, \ldots, \omega_M$

3. For each sample $\omega_j$, and given $\hat{x}(k)$, compute

   $$\lambda(k, \omega_j) = \sum_{\ell=1}^{L} \tau(\ell) \mu(k, \ell, \omega_j) - I(k)$$

4. Obtain the histogram of the distribution
Illustrative example

- 4 technologies: Nuclear, Coal, CCGT, OCGT
- Uncertainties on the load: "Wind" and "Growth"
- Sample 30,000 scenarios for the profit distribution
- *Badly represent the reality of wind (in progress)*

**Histogram of the gross margin of a CCGT unit [eur/MW]**

<table>
<thead>
<tr>
<th></th>
<th>$\hat{x}$ [MW] &quot;Wind&quot;</th>
<th>$\hat{x}$ [MW] &quot;Growth&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear</td>
<td>4797.7</td>
<td>5048.1</td>
</tr>
<tr>
<td>Coal</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>CCGT</td>
<td>4811.0</td>
<td>4269.5</td>
</tr>
<tr>
<td>OCGT</td>
<td>897.4</td>
<td>659.46</td>
</tr>
</tbody>
</table>
Illustrative example

- Risk and capital requirement:

<table>
<thead>
<tr>
<th>Gross Profit</th>
<th>”Growth”</th>
<th>”Wind”</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear : - CVaR_{30%}</td>
<td>2.1%</td>
<td>8.8%</td>
</tr>
<tr>
<td>- StDev</td>
<td>35.22</td>
<td>73.78</td>
</tr>
<tr>
<td>CCGT : - CVaR_{30%}</td>
<td>9%</td>
<td>53.5%</td>
</tr>
<tr>
<td>- StDev</td>
<td>33.6</td>
<td>86.82</td>
</tr>
<tr>
<td>OCGT : - CVaR_{30%}</td>
<td>13.4%</td>
<td>81.3%</td>
</tr>
<tr>
<td>- StDev</td>
<td>31.14</td>
<td>78.74</td>
</tr>
</tbody>
</table>

Capital requirement ≡ Capital needed for accepting the risk

- Measure by a CVaR
- Expressed in [%] of $I(k)$
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work done with A. Ehrenmann, CEEME, Gdf-Suez
E. Druenne, CEEME, Gdf-Suez
Motivations

- Power plants have quite different risk exposures
- Deterministic discount factor does not capture well the risk
- The CAPM or APT methodologies are difficult to apply
  - The market portfolio badly spans the systematic risks
  - More fundamentally: what are the systematic / idiosyncratic risks for a power company?


◊ Rather propose an approach based on risk measure
  ◊ **Risk averse** generation capacity expansion models
  ◊ Focus on the good-deal (introduced by Cochrane and Saa-Requejo)
Risk measures in short

Goals:

- Quantification of how risk affects the value of a portfolio
- Defined as capital requirement: minimal amount of capital which when added makes the position acceptable.
- For a random profit $Z$, $\rho(Z) + Z$ is acceptable
  - e.g: Mean-Variance, Exponential utility, CVaR,...

Received a lot of attention in the past decade:

- Desirable properties: axiom of coherence (by Artzner et al.)
  \textbf{Theorem} Any coherent risk measure has a dual representation:

$$\rho(Z) = \sup_{Q \in \mathcal{M}} \{\mathbb{E}_Q[-Z]\}$$

- Linked to ambiguity in economics
Risk-averse stochastic programming

Risk averse capacity expansion problem

\[
\min_{x \geq 0} \sum_{k=1}^{K} I(k)x(k) + \rho \left( -Q(x, \omega) \right)
\]

- Invest in a safe way to avoid large costs (unsatisfied demand, technology spillover,...)

- Interpretation :
  - A social planner (least cost for society)
  - An equilibrium in a complete market? (work in progress)

D. Ralph and Y. Smeers, 2010: "Pricing risk under risk measures: an introduction to stochastic-endogenous equilibria"

- Initial step to a full equilibrium model with risk averse investors
The good-deal risk measure (1)

The good-deal risk measure (from Cochrane and Saá-Requejo, 2000):

\[
\rho^\text{GD}(Z) = \sup_Q \mathbb{E}_Q \left[ -Z(\omega) \right]
\]

\[
\mathbb{E}_Q \left[ f_1(i, \omega) \right] = f_0(i)
\]

\[
\mathbb{E}_P \left[ \frac{dQ}{dP} (\omega) \right]^2 \leq \frac{1+h^2}{(R^f)^2}
\]

- A coherent risk measure in the sense of Artzner et al. (1999)
- Lead to a "risk-neutral" probability \(\rightarrow\) no arbitrage
- Incorporate the price of related risk factor \(f \rightarrow\) CAPM, APT
- Parameter \(h\) is linked to the admissible Sharpe ratio
The good-deal risk measure (2)

Problem The Lagrangian dual formulation of the good-deal:

\[ \rho^{GD}(Z) = \inf_{w, \eta} A \left( \mathbb{E}_P[\eta^2] \right)^{\frac{1}{2}} - \sum_i f_0(i)w(i) \]

\[ \eta(\omega) \geq \sum_i f_1(i, \omega)w(i) - Z(\omega) \]

Interpretation: replicating portfolio in an incomplete market

- Optimality conditions give

\[ \eta(\omega) = \max \left( 0, \sum_i f_1(i, \omega)w(i) - Z(\omega) \right) \]

- \( \eta \) measures the hedge, is called a regret

- When \( A \to \infty \): super-hedge (the value of \( Z \) is the value of a worser portfolio)
Risk-averse stochastic capacity expansion

Using the dual formulation of the good-deal measure:

$$\min_{u, w, \eta} \sum_{k=1}^{K} I(k)x(k) + w_0(i) + A\mathbb{E}[\eta(\omega)^2]^{\frac{1}{2}}$$

s.t. $x(k) \geq 0$

$0 \leq y(k, \ell, \omega) \leq x(k)$

$$\sum_{k \in K} y(k, \ell, \omega) + z(\ell, \omega) - d(\ell, \omega) \geq 0$$

$$(R_f)w + \eta(\omega) \geq \sum_{\ell=1}^{L} \tau_{\ell} \left( \sum_{k=1}^{K} c(k, \omega)y(k, \ell, \omega) + PC z(\ell, \omega) \right)$$

- Second Order Cone Program (efficient solver)
- Extension to multistage problems: time consistent
- Current work: investments that mitigate the risk due to the electrical grid, high learning rates and technological acceptance
Illustrative example

- Three stages example: 1/25/125
- Three technologies: coal, CCGT, OCGT
- 5 years time period and 2 risk factors

<table>
<thead>
<tr>
<th>α:</th>
<th>- 5%</th>
<th>0%</th>
<th>5%</th>
<th>10%</th>
<th>15%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.15</td>
<td>0.30</td>
<td>0.25</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Demand Growth

<table>
<thead>
<tr>
<th>β:</th>
<th>0%</th>
<th>12%</th>
<th>24%</th>
<th>36%</th>
<th>48%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Probability</td>
<td>0.1</td>
<td>0.4</td>
<td>0.2</td>
<td>0.25</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Gas price growth

- PC = 500 eur/Mwh
- $R_f = 1.02$ (for computing annual cost)
- $A^2 = 1.5$ ($A^2 = 1 + \text{Sharpe ratio}^2$)
Illustrative example: investment

Investment [GW] | Coal | CCGT | OCGT
---|---|---|---
$x_0(k, \omega_0)$ | 66 | 3 | 23
$E[x_1(k, \omega_1) | F_0]$ | 4.4 | 0.6 | 1.4

Impact of risk aversion:

<table>
<thead>
<tr>
<th>A=1.05</th>
<th>A=1.22</th>
<th>A=2.23</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coal</td>
<td>63</td>
<td>65</td>
</tr>
<tr>
<td>CCGT</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>OCGT</td>
<td>19</td>
<td>23</td>
</tr>
</tbody>
</table>

Initial investment ($t = 0$) for different Sharpe ratio