RISK AND RETURN IN ENVIRONMENTAL ECONOMICS

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- Costly abatement would reduce GHG emissions now, and yield uncertain future benefits.
- How important is reducing risk vs. expected benefits?
- Two related questions:

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- Iso-WTP curve is social risk-return indifference curve. For a given WTP, it describes "demand-side" policy tradeoff between risk and return.

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- Set $C_0 = 1$.

Welfare Measure

- CRRA utility. So at $t = 0$, welfare (under BAU) is:

$$W_0 = \frac{1}{1-\eta}\mathcal{E}_0 \int_0^\infty C_t^{1-\eta} e^{-\delta t} dt \ . \tag{5}$$

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- Denote $F(C, g, 0) = \mathcal{E}_0(C_t^{1-\eta})$, for $t > 0$. Write and solve Kolmogorov eqn. for F. (See paper.) Get:

$$\mathcal{E}_0(C_t^{1-\eta})e^{-\delta t} = e^{-\delta t + (1-\eta)g_0 t - \frac{1}{2}\alpha(1-\eta)t^2 + \frac{1}{6}\sigma^2(1-\eta)^2 t^3} \ . \tag{6}$$

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- As t increases, $\mathcal{E}_0(C_t^{1-\eta})$ first decreases and then increases without bound, so welfare integral must cover a finite horizon.

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- Growth rate is $g(s) = g_0 - \alpha s - \sigma \int_0^s dz = g_0 - \alpha s - \sigma z(s)$, *so*

$$C_T^{1-\eta} \ = \ e^{g_0 T - \frac{1}{2}(1-\eta)\alpha T^2 - \sigma(1-\eta)\int_0^T z(s)ds} \ , \tag{9}$$

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$$r_T = \tfrac{1}{2} T^2 C_T^{1-\eta} e^{-\delta T} \ . \tag{10}$$

Risk/Return (Two Periods)

- Want expectation and variance of this return. Expected return is: $r_T^e = \frac{1}{2}T^2\mathcal{E}_0(C_T^{1-\eta})e^{-\delta T} = \frac{1}{2}T^2 e^{-\rho_0 T - \frac{1}{2}\alpha(1-\eta)T^2 + \frac{1}{6}\sigma^2(1-\eta)^2 T^3}$ where $\rho_0 = \delta + (\eta - 1)g_0$.

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- Variance of return is:

$$\begin{aligned}\mathcal{V}(r_T) &= \mathcal{E}_0(r_T^2) - (r^e_T)^2 \\ &= \frac{1}{4}T^4 e^{-2\rho_0 T - \alpha(1-\eta)T^2 + \frac{1}{3}\sigma^2(1-\eta)^2T^3}\left[e^{\frac{1}{3}\sigma^2(1-\eta)^2T^3} - 1\right]\end{aligned}$$

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  - Note S_T doesn't depend on α. Both r_T^e and SD(r_T) grow by factor $\frac{1}{2}\alpha(1-\eta)T^2$, so this cancels out of the ratio.
  - Also $S_T \longrightarrow 0$ as $T \longrightarrow \infty$. Reason: SD(r_T) grows faster than r_T^e.

Risk/Return – Continuous Time

- Now welfare is

$$W = \frac{1}{1-\eta}\int_0^T C_t^{1-\eta} e^{-\delta t} dt$$

and return from small increase in α is:

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- Variance is $\mathcal{V}(r) = \mathcal{E}_0(r^2) - (r^e)^2$, so we need to find

$$\mathcal{E}_0(r^2) = \mathcal{E}_0\left(\int_0^T \tfrac{1}{2} t^2 C_t^{1-\eta} e^{-\delta t} dt\right)^2$$

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    - 5% prob of $T \geq 7°$C implies $\sigma_T = .242$, so $\sigma = .000061$.
    - 5% prob of $T \geq 10°$C implies $\sigma_T = .424$, so $\sigma = .000106$.

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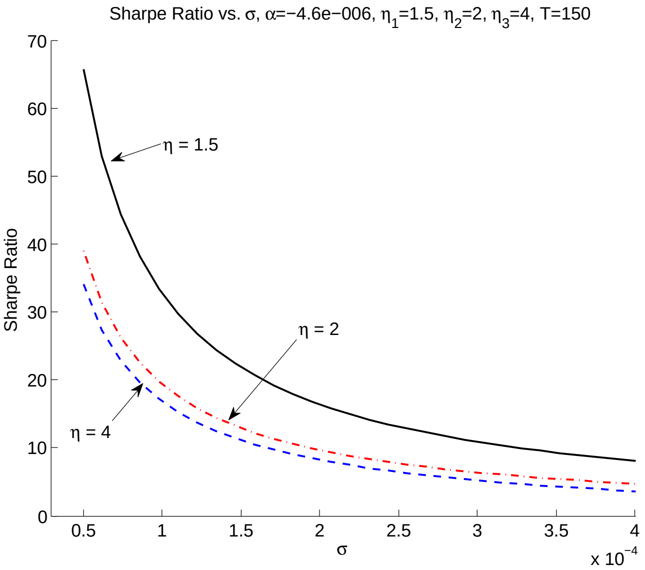
  - Expected loss of GDP if $T = 4°$C is 1% to 5%. I use 5%, which implies $\gamma = .00025$.
  - $\mathcal{E}(T) = 3°$C in 2100 implies $\alpha_T = .03$, so $\alpha = .0000075$.
  - 5% prob of $T \geq 7°$C implies $\sigma_T = .242$, so $\sigma = .000061$.
  - 5% prob of $T \geq 10°$C implies $\sigma_T = .424$, so $\sigma = .000106$.
  - I set $g_0 = .02$, $\delta = 0$, and $\eta = 2$ to 4. Also, $T = 150$ years.

Sharpe Ratio vs. σ. ($g_0 = .02$, $\delta = 0$)



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$$W_1 = \frac{1}{1-\eta}\int_0^\infty (1 - w_1)^{1-\eta} e^{-\rho_0 t + a(\alpha_1, \sigma_1, t)} dt \; . \tag{15}$$

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$$w_1 = 1 - \left[\frac{G(\alpha_1, \sigma_1)}{G(\alpha_0, \sigma_0)} \right]^{\frac{1}{\eta - 1}} , \qquad (16)$$

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- So given starting values of α and σ we can calculate WTP to decrease α and/or decrease σ.

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- Can also obtain combinations of α' and σ' for which WTP equals some arbitrary number, w. From eqn. (16), find combinations that satisfy

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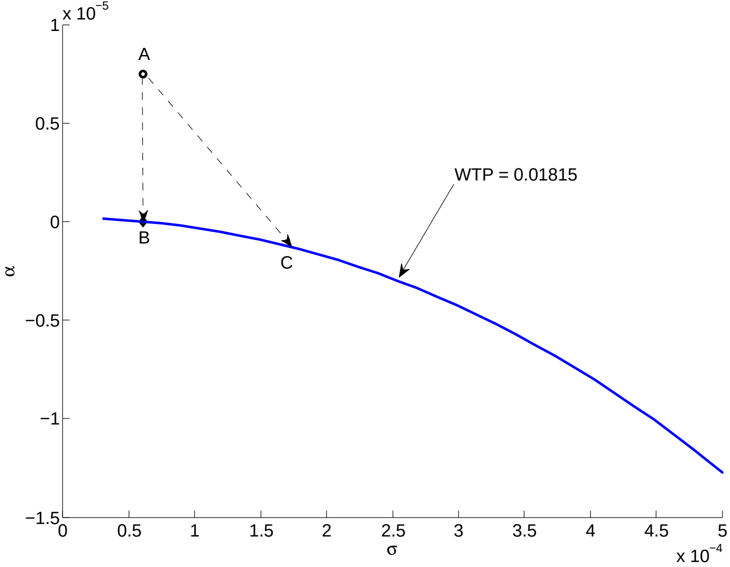
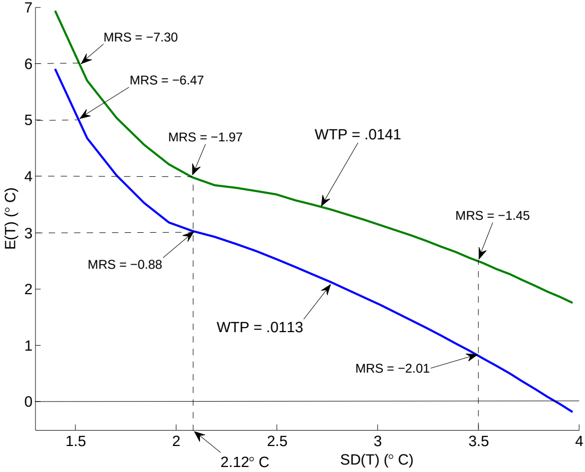
  - Curve shows other combinations of α and σ that (relative to starting values α_0 and σ_0) also have WTP of .01815.

Iso-WTP Curve ($\eta = 2$, $g_0 = .02$, $\delta = 0$)



Iso-WTP in "Uncertain Outcomes..."

Different because in that model starting point varies along curve. Ending point is $E(T) = 0$.



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- Cost of reducing α or σ? If linear or convex, can determine optimal risk-return policy mix.