Generalized Nash Equilibrium and incomplete energy markets: Market Coupling in the European Power System

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FEEM
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The context: market integration in the European Community
Market coupling as a market design:
Implemented in electricity since 2006 (in discussion for gas these days (e.g. Florence 11 March 2011))

- Day ahead and real time markets:
  1. Market coupling deals with the day-ahead market;
  2. Real time is seen as a deviation management mechanism and agents are incentivised not to resort to it.

- Integration of energy and transmission:
  1. Market coupling partially integrates energy and transmission;
  2. The zonal energy market clears on an ATC (available transmission capacity) representation of the network;
  3. Counter-trading, if necessary, takes care of the real network.

- Counter-trading:
  1. Counter-trading is operated by zonal System Operators;
  2. Without clear indication on how they coordinate.
Organisation of Cross-zonal Trade of Electricity

The energy market

- Two groups of agents:
  1. **Zonal (national) Power Exchanges (PXs)** that clear the intra and inter zone energy markets;
  2. **Zonal (national) Transmission System Operators (TSOs)** that guarantee the security of the transmission system.

**Market coupling** concentrates on the energy market and is organized as follows:

- TSOs provide the energy market with a simplified representation of the grid (today the ATC);
- PXs jointly clear the energy markets taking into account the ATC received from the TSOs;
- PXs find the equilibrium electricity quantities and prices;
- In presence of saturation of ATCs, electricity prices differ per zone.
THE CONTEXT: A contribution to the never ending debate between zonal and nodal systems:

- first in electricity, resolved in favour of nodal in the US, good arguments for zonal in EU.
- beginning in gas in EU again (point to point transmission rights will be illegal in EU.)
- in economic terms: what is the impact of an incomplete pricing of transmission.

THE TALK: The flows resulting from the PXs’ market clearing may not be feasible for the grid:

- TSOs restore feasibility by buying and selling incremental or decremental injections at the different nodes, while maintaining the zonal electricity demand and production levels unchanged. They socialized the cost of that activity. They can do that with different degree of coordination.

Assess the impact of this incomplete pricing of transmission services?
Outline

1. Methodological discussion: Generalized Nash Equilibrium and market incompleteness

2. A prototype case study

3. Conclusion.
Methodological discussion, Generalized Nash Equilibrium and market incompleteness
The illustrative example (1)
Six Node Market

The illustrative example (2)

DATA

• Demand and cost functions

<table>
<thead>
<tr>
<th>Node</th>
<th>Function Type</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Marginal Cost</td>
<td>10 + 0.05q</td>
</tr>
<tr>
<td>2</td>
<td>Marginal Cost</td>
<td>15 + 0.05q</td>
</tr>
<tr>
<td>3</td>
<td>Inverse Demand</td>
<td>37.5 - 0.05q</td>
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<tr>
<td>4</td>
<td>Marginal Cost</td>
<td>42.5 + 0.025q</td>
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<tr>
<td>5</td>
<td>Inverse Demand</td>
<td>75 - 0.1q</td>
</tr>
<tr>
<td>6</td>
<td>Inverse Demand</td>
<td>80 - 0.1q</td>
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</tbody>
</table>

The illustrative Example (3)

DATA

- **PTDF Matrix**

<table>
<thead>
<tr>
<th>Power (1 MW) Injected at Node</th>
<th>Power flow on link 1 → 6 (MW)</th>
<th>Power flow on link 2 → 5 (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.625</td>
<td>0.375</td>
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<tr>
<td>2</td>
<td>0.5</td>
<td>0.5</td>
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<tr>
<td>3</td>
<td>0.5625</td>
<td>0.4375</td>
</tr>
<tr>
<td>4</td>
<td>0.0625</td>
<td>-0.0625</td>
</tr>
<tr>
<td>5</td>
<td>0.125</td>
<td>-0.125</td>
</tr>
<tr>
<td>6 (hub)</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Capacity link 1 → 6 = 200 MW

Capacity link 2 → 5 = 250 MW

The illustrative Example (4)

Notation

- **Sets**
  - \( l = (1 - 6), (2 - 5) \): Set of lines of the transmission grid;
  - \( n = 1, \ldots, 6 \): Set of nodes;
  - \( i = 1, 2, 4 \): Subset of production nodes;
  - \( j = 3, 5, 6 \): Subset of consumption nodes

- **Variables and Parameters**
  - \( \bar{I} \): Imports/export limits among zones;
  - \( PTDF_{n,l} \): Power Transfer Distribution Factor matrix of node \( n \) on line \( l \);
  - \( \bar{F}_l \): Limit of flow through line \( l \);
  - \( q_n \): Power generated or consumed in node \( n \);
  - \( c(q_i) \): Cost function of generator located in node \( i \);
  - \( w(q_j) \): Inverse demand function of consumer located in node \( j \);
  - \( I \): Imports/exports in the PX’s problem;
  - \( \lambda^+_{l}, \lambda^-_{l} \): Marginal value of the interconnection line \( l \);
  - \( \Delta q^N_S \): Demand and generation variations (counter-trading services) in node \( n \) operated by TSO\(^N\) or TSO\(^S\)

**NOTE**: The red lines (1-6) and (2-5) have limited capacity.
1. Full integration of energy and transmission markets: the reference nodal system (Model 1)

2. Imperfect integration of energy and transmission markets: Market Coupling and centralized Counter-Trading (Model 2)

3. Imperfect integration of energy and transmission markets: Market Coupling and decentralized Counter-Trading (Model 3)
The complete market: the nodal model

The reference model:

$$\text{Min}_{q_n} \sum_{i=1,2,4} \int_0^{q_i} c_i(\xi) d\xi - \sum_{j=3,5,6} \int_0^{q_j} w_j(\xi) d\xi$$

s.t.

$$\bar{F}_l - ( \sum_{i=1,2,4} PTDF_{i,l} q_i - \sum_{j=3,5,6} PTDF_{j,l} q_j ) \geq 0 \quad (\lambda^+_l)$$

$$\bar{F}_l + ( \sum_{i=1,2,4} PTDF_{i,l} q_i - \sum_{j=3,5,6} PTDF_{j,l} q_j ) \geq 0 \quad (\lambda^-_l)$$

where \( l = (1 - 6), (2 - 5) \)

$$\sum_{i=1,2,4} q_i - \sum_{j=3,5,6} q_j = 0 \quad (\gamma)$$

$$q_n \geq 0 \quad \forall n \quad (\nu_n)$$
Results

- **Welfare**
  
  Social welfare: 23,000 €

- **Congested line**

  Both lines (1-6) and (2-5) transfer 200 MW of energy.
  Line (1-6) is congested and its marginal value is 40 €/MWh.
Market Coupling and counter-trading
The market is subdivided into two zones (North and South), each controlled by a TSO (here a 3/3 case):

The TSOs compute the ATC between the two zones.
A second example: the 4/2 case

An alternative zonal organization:
Market coupling: clearing the energy market (depends on zonal decomposition)

The PXs solve the following problem for the 3/3 configuration:

\[
\text{Min}_{q_n} \sum_{i=1,2,4} \int_0^{q_i} (\alpha + c_i(\xi))d\xi - \sum_{j=3,5,6} \int_0^{q_j} w_j(\xi)d\xi
\]

s.t.

\[
q_1 + q_2 - q_3 - I = 0
\]
\[
q_4 - q_5 - q_6 + I = 0
\]
\[
q_n \geq 0 \quad \forall n
\]
\[
-\bar{I} \leq I \leq \bar{I}
\]
Market coupling: results of the clearing of the energy market (1)

- **Welfare before re-dispatching costs**
  Welfare: 24,146 € (compared to 23,000 € in the nodal model)

- **Demand and generation**
  Total demand is 800 as in the nodal system.

- **Interconnecting line**
  The interconnection is saturated. Its marginal value is 18.33 €/MWh and the import/export is 450 MWh from North to South.
Market Coupling: optimal cross-border Counter-Trading

\[
\text{Min} \Delta q_n \quad \sum_{i=1,2,4} \int_{q_i}^{q_i + \Delta q_i} c_i(\xi) d\xi - \sum_{j=3,5,6} \int_{q_j}^{q_j + \Delta q_j} w_j(\xi) d\xi
\]

s.t.

\[-F_l \leq \sum_{i=1,2,4} (PTDF_{i,l}(q_i + \Delta q_i)) - \sum_{j=3,5,6} (PTDF_{j,l}(q_j + \Delta q_j)) \leq F_l \quad (\lambda^\pm_l)\]

where \( l = (1 - 6), (2 - 5) \)

\[
\sum_{i=1,2,4} \Delta q_i + \sum_{j=3,5,6} \Delta q_j = 0 \quad (\mu^1)
\]

\[
\sum_{i=1,2,4} \Delta q_i - \sum_{j=3,5,6} \Delta q_j = 0 \quad (\mu^2)
\]

\[
q_n + \Delta q_n \geq 0 \quad \forall n \quad (\nu_n)
\]
Computation of optimized cross-border counter-trading costs

- **Total re-dispatching cost at equilibrium**
  The total re-dispatching cost is 1,146 €

- **Average re-dispatching costs**
  The average re-dispatching cost is 1.43 €/MWh

- **Net Welfare**
  Net welfare is 23,000 €;

- **Welfare loss**
  Welfare loss is 0 € w.r.t. to Model 1 (23,000 €)
The average re-dispatching/counter-trading cost $\alpha$ incurred by the TSOs is:

$$\alpha = \frac{\sum_{i=1,2,4} \int_{q_i}^{q_i + \Delta q_i} c_i(\xi) d\xi - \sum_{j=3,5,6} \int_{q_j}^{q_j + \Delta q_j} w_j(\xi) d\xi}{\sum_{i=1,2,4} q_i}$$

This cost is charged to the users of the system. One assumes here that it is paid by the generators selling on the PXs (this is a stylized view of the problem):

**Formulation**: Suppose that the average counter-dispatching cost adds to the constant term (10, 15 and 42.5) of the marginal cost functions

- The fixed point problem can be solved by looping between the PX and TSO models
- Alternative arrangements can have counter-trading paid by demand nodes or by both demand and supply nodes.
- From here on we only report the results at equilibrium between the PX (market coupling) and the TSO (counter-trading)
## Results of optimal cross-border counter-trading (2)

<table>
<thead>
<tr>
<th></th>
<th>Configuration 3/3</th>
<th></th>
<th>Configuration 4/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prod/Cons</td>
<td>Prod</td>
<td>Prod/Cons</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>30</td>
<td>125</td>
<td>8</td>
</tr>
<tr>
<td>TRC</td>
<td>1,146</td>
<td>1,235</td>
<td>5,079</td>
</tr>
<tr>
<td>ARC</td>
<td>1.51</td>
<td>1.63</td>
<td>6.51</td>
</tr>
<tr>
<td>Net welfare</td>
<td>22,970</td>
<td>22,875</td>
<td>22,992</td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>40.00</td>
<td>42.22</td>
<td>40.00</td>
</tr>
<tr>
<td>MV line (2-5)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**NOTATION:**

- Welfare loss (€) are computed w.r.t. Model 1 (23,000 €);
- TRC: Total Re-dispatching Costs in €;
- ARC: Average Re-dispatching Costs in €/MWh;
- Net Welfare: welfare value at equilibrium in €;
- MV: marginal value of transmission lines in €/MWh
Market coupling: counter-trading with different degrees of coordination

- The law (third package) says that TSOs must coordinate
  - But it does not say how

- EU documents do not envisage an optimal cross border counter-trading
  - (e.g. impact assessments of infrastructure document of November 2010)

- It thus makes sense to make assumptions on lack of optimal cross border counter-trading

- National TSOs are not coordinated (Model 3):
  A. National TSOs have full access to all re-dispatching resources: an internal market of counter-trading resources (model 3.1);
  B. National TSOs have only a limited access to all re-dispatching resources: a limited internal market of counter-trading resources (model 3.2);
  C. National TSOs manage only the re-dispatching resources in their control area: national markets of counter-trading resources (model 3.3)
Imperfect counter-trading
Technical note: computation and economics

1. Nodal pricing: optimization;
2. Energy market clearing and optimized cross-border counter-trading: sequence of optimization or complementarity problems;
Technical note: completing the transmission market

1. Access to counter-trading resources: create an internal market of counter-trading resources (by (close) analogy with discussion on internal market of balancing resources);

2. Create a market of interconnection line capacity (by analogy with PJM MISO interconnection);

3. Merger or coordination contracts between TSOs (as taking place on the market).
MODEL 3.1 an internal market of counter-trading resources; no market of line capacity (1)

Assume that each TSO can buy counter-trading services in both zones. Note \( \Delta q_i^{N,S} \) where \( i = 1, 2, \ldots, 6 \) these actions of both TSOs.

- **TSO\(^N\) (with a similar problem for TSO\(^S\))**

\[
\text{Min} \Delta q_i^N \quad \sum_{i=1,2,4} \int_{q_i+\Delta q_i^S}^{q_i+\Delta q_i^N} c_i(\xi) \, d\xi - \sum_{j=3,5,6} \int_{q_j+\Delta q_j^S}^{q_j+\Delta q_j^N} w_j(\xi) \, d\xi
\]

\[\text{s.t.} \]

\[\overline{F}_l - \left( \sum_{i=1,2,4} \text{PTDF}_{i,l}(q_i+\Delta q_i^N+\Delta q_i^S) - \sum_{i=3,5,6} \text{PTDF}_{i,l}(q_i+\Delta q_i^N+\Delta q_i^S) \right) \geq 0 \quad (\lambda_l^{N,+})\]

where \( l = (1-6), (2-5) \)

\[
\sum_{i=1,2,4} \Delta q_i^N + \sum_{j=4,5,6} \Delta q_j^N = 0 \quad (\mu_l^{N,1})
\]

\[
\sum_{i=1,2,4} \Delta q_i^N - \sum_{j=4,5,6} \Delta q_j^N = 0 \quad (\mu_l^{N,2})
\]

\[q_n + \Delta q_n^N + \Delta q_n^S \geq 0 \quad \forall n \quad (\nu_n^N)\]
MODEL 3.1 An internal market of counter-trading resources; no market of line capacity (2)

Assume that \( q_n + \Delta q_n^N + \Delta q_n^S > 0 \) for \( \forall n \) and define the dual variables \( \lambda^N_l = (-\lambda^N_l^+, + \lambda^N_l^-) \) for \( l = ((1 - 6), (2 - 5)) \). We obtain the dual conditions:

\[
c_i - \sum_l \lambda^N_l \cdot PTDF_{i,l} - \mu^N,1 + \mu^N,2 = 0 \quad i = 1, 2, 4
\]

\[-w_j + \sum_l \lambda^N_l \cdot PTDF_{j,l} - \mu^N,1 - \mu^N,2 = 0 \quad j = 3, 5, 6
\]

Writing the same relations for \( TSO^S \)

\[
c_i - \sum_l \lambda^S_l \cdot PTDF_{i,l} - \mu^S,1 + \mu^S,2 = 0 \quad i = 1, 2, 4
\]

\[-w_j + \sum_l \lambda^S_l \cdot PTDF_{j,l} - \mu^S,1 - \mu^S,2 = 0 \quad j = 3, 5, 6
\]

we find, if there are enough equalities (here six equalities for four variables), that \( \lambda^N_l = \lambda^S_l \).
MODEL 3.1 An internal market of counter-trading resources; no market of line capacity (3)

**PROPOSITION**: The internal market of counter-trading resources restores the perfect counter-trading: (i) different agents resorting to the same counter-trading resource at the same price induces a price arbitrage that forces the equality of the dual variables of the common constraints (the transmission lines (ii) which has an effect equivalent to a market of line capacity) and hence (iii) leads to a single GNE in counter-trading. (iv) In mathematical terms, the solution set of the QVI is identical to the solution set of the associated VI!!
The Nabetani, Tseng, Fukushima’s parametrized approach is as follows:

\[
\begin{align*}
\text{Min} & \quad \Delta q^N_n, S_n \\
\sum_{i=1, 2, 4} & \int q_i + \Delta q^S_i + \Delta q^N_i c_i(\xi) d\xi - \sum_{j=3, 5, 6} \int q_j + \Delta q^S_j + \Delta q^N_j w_j(\xi) d\xi + \\
& + \sum_l \left( \gamma^N_{l^+} - \gamma^N_{l^-} \right) \cdot \left( \sum_{i=1, 2, 4} \text{PTDF}_{i,l} \cdot \Delta q^N_i - \sum_{j=3, 5, 6} \text{PTDF}_{j,l} \cdot \Delta q^N_j \right) \\
& + \sum_l \left( \gamma^S_{l^+} - \gamma^S_{l^-} \right) \cdot \left( \sum_{i=1, 2, 4} \text{PTDF}_{i,l} \cdot \Delta q^S_i - \sum_{j=3, 5, 6} \text{PTDF}_{j,l} \cdot \Delta q^S_j \right) \\
\text{s.t.} & \quad F_l - \left( \sum_{i=1, 2, 4} \text{PTDF}_{i,l}(q_i + \Delta q^N_i + \Delta q^S_i) - \sum_{j=3, 5, 6} \text{PTDF}_{j,l}(q_j + \Delta q^N_j + \Delta q^S_j) \right) \geq 0 \quad (\lambda^+_l) \\
\sum_{i=1, 2, 4} & \Delta q^Z_i + \sum_{j=3, 5, 6} \Delta q^Z_j = 0 \quad Z = N, S \quad (\mu^Z_{l^1}) \\
\sum_{i=1, 2, 4} & \Delta q^Z_i + \sum_{j=3, 5, 6} \Delta q^Z_j = 0 \quad Z = N, S \quad (\mu^Z_{l^2}) \\
q_n + \Delta q^N_n + \Delta q^S_n & \geq 0 \quad \forall n \quad (\nu^Z_n) \quad Z = N, S
\end{align*}
\]
MODEL 3.2: A restricted internal market of counter-trading resources

Each TSO can buy counter-trading services in both zones, but purchase in other zone is limited.

- **TSO\(^N\)** (with a similar problem for TSO\(^S\))

\[
\text{Min}_{q_n^N} \sum_{i=1,2,4} \int q_i + \Delta q_i^N + \Delta q_i^S c_i(\xi) d\xi - \sum_{j=3,5,6} \int q_j + \Delta q_j^N + \Delta q_j^S w_j(\xi) d\xi \quad \text{s.t.}
\]

\[
\bar{F}_l - (\sum_{i=1,2,4} PTDF_{i,l}(q_i + \Delta q_i^N + \Delta q_i^S) - \sum_{j=3,5,6} PTDF_{j,l}(q_j + \Delta q_j^N + \Delta q_j^S)) \geq 0 \quad (\lambda_l^{N,+})
\]

where \(l = (1 - 6), (2 - 5)\)

\[
\sum_{i=1,2,4} \Delta q_i^N + \sum_{j=3,5,6} \Delta q_j^N = 0 \quad (\mu_l^{N,1})
\]

\[
\sum_{i=1,2,4} \Delta q_i^N - \sum_{j=3,5,6} \Delta q_j^N = 0 = 0 \quad (\mu_l^{N,2})
\]

\[
q_n + \Delta q_n^N + \Delta q_n^S \geq 0 \quad \forall n \quad (\nu_n^N)
\]

\[
-\Delta q_n^N \leq \Delta q_n^N \leq \Delta q_n^N \quad n = 4, 5, 6 \quad (\eta_n^{N,\pm})
\]
A FIRST RESULT: The implicit market of interconnection line capacity is lost when TSOs have limited access to counter-trading resources in other zones. This introduces inefficiencies; these can be more or less important depending on whether one does or does not introduce a market for transmission.

MAIN QUESTION: How far can one go in deteriorating the efficiency of counter-trading and hence the overall efficiency of market coupling?
Results of Model 3.2 (1)
Market configuration 3/3 with a market of interconnection capacity

Trading interconnection capacities among TSOs (setting $\gamma_{l}^{N/S,\pm} = 0$) under the following counter-trading resource restrictions, counter-trading remains relatively efficient:

<table>
<thead>
<tr>
<th>$\Delta q_{1}^{S}$</th>
<th>$\Delta q_{2}^{S}$</th>
<th>$\Delta q_{3}^{S}$</th>
<th>$\Delta q_{4}^{N}$</th>
<th>$\Delta q_{5}^{N}$</th>
<th>$\Delta q_{6}^{N}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.33</td>
<td>16.67</td>
<td>8.33</td>
<td>16.67</td>
<td>8.33</td>
<td>16.67</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Configuration 3/3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prod/Cons</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>30</td>
</tr>
<tr>
<td>TRC</td>
<td>1,146</td>
</tr>
<tr>
<td>ARC</td>
<td>1.51</td>
</tr>
<tr>
<td>Net welfare</td>
<td>22,970</td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>40.00</td>
</tr>
<tr>
<td>MV line (2-5)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma_{l}^{N/S,\pm}$</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Results of Model 3.2 (2)

Market configuration 3/3 without market of line capacity

Eliminating trade of transmission capacities (setting $\gamma_{(1-6)}^{N,+} = 60$ and $\gamma_{(1-6)}^{S,+} = 0$) and keeping the same limits on counter-trading resources, restrictions degrades the efficiency of counter-trading:

<table>
<thead>
<tr>
<th></th>
<th>Configuration 3/3</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Prod/Cons</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>396</td>
</tr>
<tr>
<td>TRC</td>
<td>1,490</td>
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<tr>
<td>ARC</td>
<td>2</td>
</tr>
<tr>
<td>Net welfare</td>
<td>22,604</td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>22.67</td>
</tr>
<tr>
<td>MV line (2-5)</td>
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</tr>
<tr>
<td>$\gamma_{(1-6)}^{N,+}$</td>
<td>60.00</td>
</tr>
<tr>
<td>$\gamma_{(1-6)}^{S,+}$</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Results of Model 3.2 (3)
Market configuration 4/2 with a market of line capacity

Setting $\gamma_l^{N/S,\pm} = 0$ under the following counter-trading resource restrictions, the result can be very bad depending on how one organizes counter-trading, even in presence of a transmission market:

<table>
<thead>
<tr>
<th>$\Delta q_1^S$</th>
<th>$\Delta q_2^S$</th>
<th>$\Delta q_3^S$</th>
<th>$\Delta q_4^N$</th>
<th>$\Delta q_5^N$</th>
<th>$\Delta q_6^S$</th>
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<tbody>
<tr>
<td>72.48</td>
<td>22.48</td>
<td>94.96</td>
<td>94.96</td>
<td>34.98</td>
<td>59.98</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Configuration 4/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prod/Cons</td>
<td>Prod</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>8</td>
</tr>
<tr>
<td>TRC</td>
<td>5,079</td>
</tr>
<tr>
<td>ARC</td>
<td>6.51</td>
</tr>
<tr>
<td>Net welfare</td>
<td>22,992</td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>40.00</td>
</tr>
<tr>
<td>MV line (2-5)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma_l^{N/S,\pm}$</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Setting $\gamma_{(1-6)}^N = 60$ and $\gamma_{(1-6)}^S = 0$ and keeping the same limits on counter-trading resources restrictions, one has for configuration 4/2:

<table>
<thead>
<tr>
<th></th>
<th>Configuration 4/2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prod/Cons</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>2,472</td>
</tr>
<tr>
<td>TRC</td>
<td>7,191</td>
</tr>
<tr>
<td>ARC</td>
<td>9.76</td>
</tr>
<tr>
<td>Net welfare</td>
<td>20,528</td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>18.16</td>
</tr>
<tr>
<td>MV line (2-5)</td>
<td>0.00</td>
</tr>
<tr>
<td>$\gamma_{(1-6)}^N$</td>
<td>60.00</td>
</tr>
<tr>
<td>$\gamma_{(1-6)}^S$</td>
<td>0.00</td>
</tr>
</tbody>
</table>
A SECOND RESULT: Not only the implicit coordination is lost when TSOs have limited access to counter-trading resources in other zones, but counter-trading can become impossible in some zonal markets (in this case example 4/2) (as encountered in the single zone Sweden).

A THIRD RESULT: Even when counter-trading is possible (examples 3/3 and 4/2) it can become extremely inefficient in the absence of a market for transmission capacities (different $\gamma$).
Assume that each TSO can only buy counter-trading services in its zone.

- **TSO$^N$ (similar problem for the TSO$^S$)**

\[
\text{Min}_{\Delta q_n^N} \sum_{i=1,2} \int_{q_i}^{q_i + \Delta q_i^N} c_i(\xi) d\xi - \int_{q_3}^{q_3 + \Delta q_3^N} w_3(\xi) d\xi \\
\text{s.t.} \\
\bar{F}_l - (\sum_{i=1,2} PTDF_{i,l}(q_i + \Delta q_i^N) + PTDF_{4,l}(q_4 + \Delta q_4^S) + \\
- PTDF_{3,l}(q_3 + \Delta q_3^N) - \sum_{j=5,6} PTDF_{j,l}(q_j + \Delta q_j^S)) \geq 0 \quad (\lambda_i^+) \\
\text{where } l = (1 - 6), (2 - 5) \\
\Delta q_1^N + \Delta q_2^N + \Delta q_3^N = 0 \quad (\mu_{l,1}^N) \\
\Delta q_3^N - \Delta q_1^N - \Delta q_2^N = 0 \quad (\mu_{l,2}^N) \\
q_n + \Delta q_n^N \geq 0 \quad n = 1, 2, 3
Only imposing that each TSO remains in balance, even when assuming a transmission market (setting all the weight $\gamma_{N/S,\pm}^i = 0$) can make the situation difficult:

<table>
<thead>
<tr>
<th></th>
<th>Configuration 3/3</th>
<th></th>
<th>Configuration 4/2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Prod/Cons</td>
<td>Prod*</td>
<td>Prod/Cons*</td>
<td>Prod</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>1,454</td>
<td>3,096</td>
<td>3,009</td>
<td>infeasible</td>
</tr>
<tr>
<td>TRC</td>
<td>2,442</td>
<td>3,781</td>
<td>7,629</td>
<td>infeasible</td>
</tr>
<tr>
<td>ARC</td>
<td>3.45</td>
<td>5.88</td>
<td>10.50</td>
<td>infeasible</td>
</tr>
<tr>
<td>Net welfare</td>
<td>21,546</td>
<td>19,904</td>
<td>19,991</td>
<td>infeasible</td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>146.67</td>
<td>220.00</td>
<td>82.30</td>
<td>infeasible</td>
</tr>
<tr>
<td>MV line (2-5)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>infeasible</td>
</tr>
<tr>
<td>$\gamma_{N/S,\pm}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>infeasible</td>
</tr>
</tbody>
</table>

* These scenarios admit always the same solution whatever $\gamma$ considered.
When setting $\gamma^{N,+}_{(1-6)} = 60$ and $\gamma^{S,+}_{(1-6)} = 0$ (eliminating the transmission market), the situation becomes dramatic:

<table>
<thead>
<tr>
<th></th>
<th>Configuration 3/3</th>
<th>Prod/Cons</th>
<th>Prod</th>
<th>Configuration 4/2</th>
<th>Prod/Cons</th>
<th>Prod</th>
</tr>
</thead>
<tbody>
<tr>
<td>Welfare loss</td>
<td>1,555</td>
<td>3,096</td>
<td>3,009</td>
<td>infeasible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TRC</td>
<td>2,529</td>
<td>3,781</td>
<td>7,629</td>
<td>infeasible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ARC</td>
<td>3.60</td>
<td>5.88</td>
<td>10.50</td>
<td>infeasible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net welfare</td>
<td>21,445</td>
<td>19,904</td>
<td>19,991</td>
<td>infeasible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MV line (1-6)</td>
<td>106.67</td>
<td>160.00</td>
<td>82.30</td>
<td>infeasible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>MV line (2-5)</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>infeasible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^{N,+}_{(1-6)}$</td>
<td>60.00</td>
<td>60.00</td>
<td>60.00</td>
<td>60.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma^{S,+}_{(1-6)}$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A GENERAL OBSERVATION: Counter-trading efficiency further and dramatically deteriorates when one completely segments the counter-trading resources market.

AN ADDITIONAL RESULT: Even when counter-trading is possible (example 3/3) it can become extremely inefficient in the absence of a market for transmission capacities (different $\gamma$).
Summing up on this example and barring incentives to game counter-trading

**Nodal pricing** is, as expected, the best system. **Integrating counter-trading services and TSOs** does well. **Integrating counter-trading resources, even without a transmission market** does as well (but requires drastic harmonization of market design between zones). **Partially segmenting the counter-trading resources, with or without a transmission market** can seriously degrade efficiency and even make counter-trading impossible. **Totally segmenting counter-trading resources, with or without a transmission market** further deteriorates efficiency.
Case Study
A toy model of Central Western European (CWE) Market:

Market coupling is currently operated among Belgium, France and the Netherlands and Germany.

**SOURCE:** Energy research Centre of the Netherlands (ECN) website
Model 1: Nodal Model
Central Western European Market

MAIN RESULTS
Considering different demand scenarios of the nodal model, we obtain the following results:

<table>
<thead>
<tr>
<th>Demand level</th>
<th>Social Welfare (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>267,124</td>
</tr>
<tr>
<td>Increase 5%</td>
<td>279,254</td>
</tr>
<tr>
<td>Increase 10%</td>
<td>291,080</td>
</tr>
<tr>
<td>Increase 20%</td>
<td>313,592</td>
</tr>
</tbody>
</table>

Table: Welfare of different nodal model scenarios
PXs solve a welfare maximization problem while taking into account the following stylized representation of the transmission network:

The social welfare resulting from the clearing of the energy market, before removing violations of line constraints, amounts to 267,571 M€.
**MAIN RESULTS**

All TSOs coordinate counter-trading. Results for different demand scenarios:

<table>
<thead>
<tr>
<th>Demand level</th>
<th>Total Re-dispatching costs (M€)</th>
<th>Average Re-dispatching costs (€/MWh)</th>
<th>Welfare (PX) (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference</td>
<td>450</td>
<td>0.37</td>
<td>267,120</td>
</tr>
<tr>
<td>Increase 5%</td>
<td>431</td>
<td>0.35</td>
<td>279,249</td>
</tr>
<tr>
<td>Increase 10%</td>
<td>550</td>
<td>0.43</td>
<td>291,066</td>
</tr>
<tr>
<td>Increase 20%</td>
<td>322</td>
<td>0.24</td>
<td>313,590</td>
</tr>
</tbody>
</table>

*Table: Welfare and re-dispatching costs*

Welfare losses respectively amount to 4, 5, 14 and 2 million €/year w.r.t. the values obtained in Models 1.
Only one TSO operates in the market. This coordinates the re-dispatching activities inside and on the interconnections of France, Belgium and the Netherlands. This market organization is depicted as follows:
MAIN RESULTS

<table>
<thead>
<tr>
<th>Total Re-dispatching costs (M€)</th>
<th>Average Re-dispatching costs (€/MWh)</th>
<th>Welfare (PX) (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>455</td>
<td>0.38</td>
<td>267,116</td>
</tr>
</tbody>
</table>

Table: Welfare and re-dispatching costs

Welfare losses amount to 4 and 8 million €/year w.r.t. the reference values obtained in Models 1 and 2 respectively.
Three TSOs operate in the market:

- (F-B-NL) TSO manages the re-dispatching activities in France, Belgium and the Netherlands;
- (G-NL) TSO manages the re-dispatching activities in Germany and in the Netherlands;
- (G-F) TSO manages the re-dispatching activities in Germany and in France.
MAIN RESULT: one restores the efficiency of an integrated counter-trading

This model creates arbitrage possibilities between TSOs that have un-discriminatory access to common counter-trading resources. This assumption allows TSOs to implicitly coordinate their action: we fall back on the results of Model 2 where we consider an explicit coordination.
Two Bilateral TSOs: Model 3.3 (1)

Uncoordinated Counter-Trading

Two TSOs operate in the market. They manage congestion on the interconnection lines between France and Belgium (note as is the case between RTE (F) and Elia (B)) and Belgium and the Netherlands (as is not the case between Elia (B) and TenneT (NL)). One is the (F-B) TSO and the other is the (B-NL) TSO. They share counter-trading resources in Belgium as illustrated in the following picture:
Two Bilateral TSOs: Model 3.3 (2)
Uncoordinated Counter-Trading

**MAIN RESULTS**

<table>
<thead>
<tr>
<th>Variation limits for (B-NL) TSO</th>
<th>Total Re-dispatching cost (M€)</th>
<th>Average Re-dispatching costs (€/MWh)</th>
<th>Welfare (PX) (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>936</td>
<td>455</td>
<td>0.38</td>
<td>267,116</td>
</tr>
<tr>
<td>936*0.5</td>
<td>455</td>
<td>0.38</td>
<td>267,116</td>
</tr>
<tr>
<td>936*0.1</td>
<td>460</td>
<td>0.38</td>
<td>267,111</td>
</tr>
</tbody>
</table>

Table: (B-NL) has limited action in Belgium: degradation with respect to Model 2

<table>
<thead>
<tr>
<th>Variation limits for (F-B) TSO</th>
<th>Total Re-dispatching cost (M€)</th>
<th>Average Re-dispatching costs (€/MWh)</th>
<th>Welfare (PX) (M€)</th>
</tr>
</thead>
<tbody>
<tr>
<td>898</td>
<td>455</td>
<td>0.38</td>
<td>267,116</td>
</tr>
<tr>
<td>898*0.5</td>
<td>455</td>
<td>0.38</td>
<td>267,116</td>
</tr>
<tr>
<td>898*0.1</td>
<td>656</td>
<td>0.55</td>
<td>266,914</td>
</tr>
</tbody>
</table>

Table: (F-B) has limited action in Belgium: degradation with respect to Model 2

Welfare losses amount to 5 and 202 million € for the cases “936*0.1” and “898*0.1”, respectively.
There are four TSOs: one per each national market. None of the TSOs controls the interconnection lines:

This problem is infeasible, but feasibility can be restored with a significant investment in the grid (in practice by reducing ATC for the PXs).
**MAIN RESULTS**

This segmentation of the TSOs’ action implies market inefficiencies as results show:

- Welfare amounts to 264,182 € (loss of 2.9 billion €/year w.r.t. the welfare of Model 1);
- High average re-dispatching costs in Belgium (4.32 €/MWh) and in the Netherlands (35.67 €/MWh);
- No re-dispatching costs in France and in Germany.
Conclusions
Conclusions

1. Counter-trading can be costly: this has indeed been observed in practice e.g. ERCOT;

2. As expected the less coordination, the more costly it can be;

3. Counter-trading can even be impossible (also observed in practice (e.g. PECO, Sweden))